Harmony

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0-Introduction

Harmony is the manifestation of notes sounded simultaneously. The theory of harmony comprehends the fundamental elements of Western music, especially the common practice period (about 1650-1900).

Music can also exist in the form of a single melody and its rhythmic organization: many musical cultures, such as those of India and China, primarily oriented their developing to the horizontal dimension of the musical sounds, the counterpoint. Counterpoint focuses on the writing of melodies, which have an inner logic in terms of some scales (also called *modes*) out of which they are formed.

During the European Middle Ages and Renaissance, however, the research of conventions and handy rules for writing choral liturgical music led musicians to the idea that melody was only the uppermost layer of a piece of music. By means of uncountable attempts and experiments, the musical scales have been synthesized into a new and profound setup, the tonality. Tonality never stopped to be the central element of the most relevant Western music up to the 20th century.

Since the harmony involves the acoustic aspects of the chords which are produced through a superposition of different melodies, counterpoint and harmony cannot be treated as separated entities. It is also to be considered that, whenever some notes are sounded consecutively, the ear creates their simultaneity, since it perceives the harmony that would be produced if the notes had sounded together.

Although the main acoustic properties of notes and chords don't change, being generally independent from the characteristics of the instruments, during the evolution of tonal music many of the early guidelines of the voice-leading, such as the treatment of dissonance, have been gradually unnoticed by the greatest composers. On the other hand, a significant amount of rules were introduced for meeting the needs of choral music, where the circumstances forced composers to obtain complexity only through the superposition of simple melodic lines. The historical development of the harmony gives help in explaining how the natural material, i.e. the notes and their harmonics, have been brought into relation with the human perception of the sounds. Therefore, this text will introduce the most recent progresses of the theory of harmony, such as the chromatic voice-leading and the so-called *vagrant harmonies*, after having discussed the traditional four-part vocal writing.

A primary role will be given to the analysis of the harmonics, or *overtones*, which are involved in the sound produced by all the musical instruments. Their presence is a fundamental aspect of the human perception of the chords and the relationships between them. The examination of the harmonics will provide some explanations to various characteristics of tonal music, such as the minor tonality and the consonance or dissonance of the intervals.

Few remarks on the historical background within which Western harmony has been developed are given in chapter 2. Furthermore, since harmony doesn't be the first subject to be examined when approaching the theory of music, a short overview of the music fundamentals and the common notation is given in chapter 3. This is nothing more than an attempt to give the text some completeness.

1-Harmonics and diatonic scale

Sounds are waves travelling through the air or some other medium. The musical notes, or *tones*, are sounds having one recognizable frequency. The frequency, also called the "pitch", is the highness of a sound. Technically, the frequency of a wave is the number of oscillations per second performed by the air molecules involved in its (mechanical) propagation, and is measured in Hertz (Hz). The oscillations' amplitude is related to the volume of the sound, and is measured in decibels (db).

Whereas the noises consist of a casual superposition of sound waves having different frequencies, a traditional musical instrument, except some percussions, is an object that produces a mixture of waves where a frequency which is much louder than the others can be detected: it is the frequency of the note that is sounded by the instrument.

An essential property of all musical instruments (as well as the vocal cords) is the following: in addition to the note's own frequency, also its multiples have considerable volumes. That is, if an instrument is playing e.g. the note A corresponding to the frequency 110 Hz, then also the frequencies:

110*2 =220 Hz, 110*3=330 Hz, 110*4=440 Hz, 110*5=550 Hz, …

are particularly emphasized. These frequencies are the **harmonics** or **overtones** of the note that is sounded. The list would be endless, but the volume of the harmonics diminishes (such decrease is non-uniform and doesn't follow any fixed law) as the frequency increases, so that only the initial harmonics can be detected. Usually the term "harmonics" comprehends the note itself, which is the 1st and loudest harmonic, while the term "overtones" refers to the rest of the harmonics. The 1st harmonic is called **fundamental frequency**.

Every instrument accentuates the multiples of the note that it's sounding, that is, instruments sounding the same note produce exactly the same list of harmonics.

The presence of the harmonics in the musical sounds is related to the mechanical properties of the instruments themselves, which are resonant objects. However, each instrument accentuates some harmonics more than others: that is, whereas the frequencies of the harmonics don't change if a different instrument is taken, their volumes are a specific aspect of each source of sound. The (infinitely populated) set of harmonics' volumes of an instrument is called **timbre**. The timbre makes a sound distinctive, so that it's possible to recognize the particular instrument that is sounding.

The differences between the timbres of different instruments can usually be neglected when writing music: the magnificence of a magnificent piece generally doesn't depend on the kind of instruments that are sounding. However, since the frequencies of the harmonics are the same for all instruments, when putting the notes together for creating chords (a **chord** is a group of more

than two notes sounded simultaneously) and melodies, the presence of the harmonics plays an essential role. The musical activities usually focus on the first 10-15 harmonics; in particular, the initial 6 multiples make the sound of the instruments particularly pleasant, and whenever higher harmonics prevail over them, the notes become harsh and dull.

Among the harmonics of a note, the 2^{nd}, 4^{th}, 8^{th}, 16^{th}, … (the powers of 2) correspond to very special harmonics: they are the **octave**s of the note. The sound produced by a note sounded along with one of its octaves is very distinctive: it's one of the most basic acoustic aspects of music.

Outside the score, the notes are indicated by the seven letters from A to G. The octaves of a note are associated to the same letter. That is, taking e.g. the note C, by multiplying its frequency by 2, 4, 8, 16, … another C is obtained. The frequency-gap between a note and its first octave (twice the note's frequency) comprehends the pitches of all the other notes.

The lowest note that is used in music is the C that corresponds to about 16.35 Hz (hardly hearable by humans). In the American Standard (or Scientific) Pitch Notation, this C is denoted as C0. The octaves of C0 are indicated with C1, C2, C3, …, that is:

C1= C0*2, C2=C0*4, C3=C0*8, C4=C0*16, …

The octaves of C0 are used as a reference list for naming all the other notes: if the frequency of a note, say G, is comprehended between e.g. C3 and C4, this G will be called G3. This method allows to recognize each G among all its octaves.

All the harmonics of a note, and not only the octaves, can be associated to other notes. Taking e.g. C2 as fundamental frequency, the initial harmonics are:

C2, **C3**, G3, **C4**, E4, G4, Bb4, **C5**, D5, E5, F#5, G5, Ab5, Bb5, B5, **C6**, …

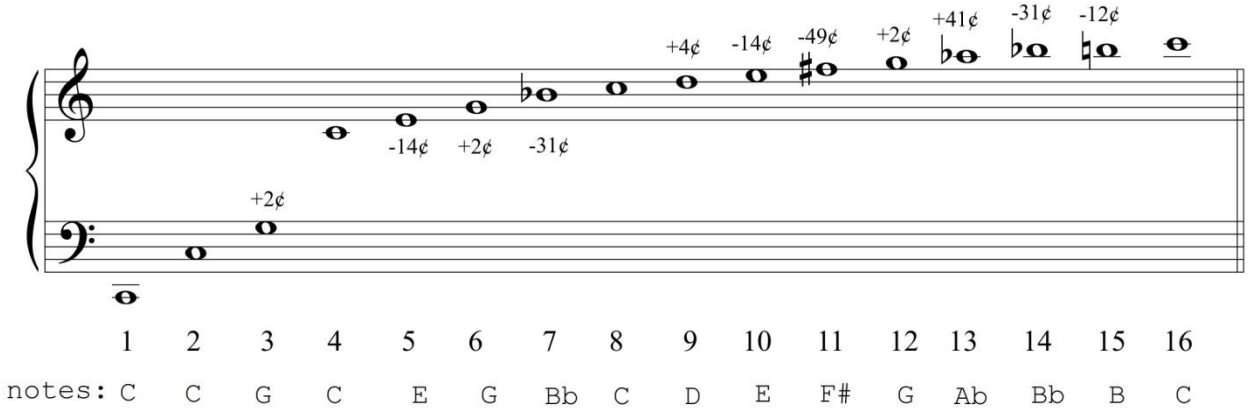
where the bolded letters mark the octaves of C. This list correspond to the frequencies:

65.4, 130.8, 196.2, 261.6, 327.0, 392.4, 457.8, … (Hz)

However, if these frequencies are compared with the above notes sounded on e.g. a traditional piano, it will be possible to observe that, except the octaves, they don't be exactly the same pitch: the harmonics and the piano's notes differ for some amount of Hz. For to assign each harmonic to a note, therefore, some approximation is required. A criterion for establishing these approximations is called **tempered system**. The **equally-tempered system** is by far the most used tempered system in Western music (and also many non-Western music).

The equally-tempered system divides the gap between every note and its first octave into 12 equally-separated pitches. To each of the pitches corresponds a note, and the closeness of the frequency of a harmonic to one of the pitches allows to assign a note to the harmonic. The gap between two consecutive pitches is called **semitone** or **half-step**, so that between a note and its first octave there are 12 semitones. Each semitone is itself subdivided into 100 cents (symbolized with ¢). Two consecutive semitones, i.e. a **whole-step** (200 ¢), are called **tone**.

In the figure 1.1, the notes corresponding to the first harmonics of C are shown on the score along with their equally-tempered system's approximations (given in cents).



[**Figure 1.1**: Harmonics of C2 and their equally-tempered system's approximations.]

The difference in pitch between two notes is called **interval**. The most important interval is between a note and its first octave, which is itself called "octave". The octave interval exactly corresponds to the distance between the 1st and the 2nd harmonics of every note, and also between the 2nd and the 4th, the 4th and the 8th, the 8th and the 16th, and so on.

Furthermore, the interval between the 2nd and the 3rd harmonics (in the above example, C3-G3) is called "perfect fifth" interval or just "fifth", and the distance between the third and the fourth harmonic (G3-C4) is the "perfect fourth" interval or "fourth". These intervals, the octave, the fifth, and the fourth, occur in the music of almost all cultures. Other common intervals can also be found in the sequence of the harmonics: e.g., the "major third" interval (between the 4th and the 5th harmonics) and the "minor third" interval (between the 5th and the 6th).

Since the intervals are defined as the "distances" between the harmonics, they don't change when a different fundamental frequency is considered: an octave always lies between the 1st and the 2nd harmonics, a perfect fifth always lies between the 2nd and the 3rd, and so on. That is, every note brings with itself the same sequence of intervals.

However, if an interval is shifted to a higher or lower pitch, then the frequency-gap it corresponds to doesn't remain the same. E.g., the note C3 (130.8 Hz) is twice the frequency of C2 (65.4 Hz), and C4 (261.6) is twice the frequency of C3: the difference in frequency between C2 and C3 is half the difference in frequency between C3 and C4, but an octave interval continues to separate C1 from C2, and C2 from C3. In other words, each octave corresponds to a frequency-gap which is twice the range of the previous octave, and which is half the range of the next. If considering the sequence of 12 consecutive semitones which the octave is subdivided into, then each semitone increases in frequency by a factor of +1.059 Hz (the twelfth root of 2) with respect to the previous (adjacent) semitone, while the tones increase by a factor of +1.122 Hz (the sixth root of 2).

Since the same interval changes its frequency-range when it is shifted higher or lower, the human brain recognizes the intervals between the notes without knowing the frequencies of the notes. This evaluation is possible because of the presence of the harmonics: e.g., if the 1^{st}, 2^{nd}, 3^{rd}, 4^{th}, … harmonics of a note respectively coincide with the 2^{nd}, 4^{th}, 6^{th}, 8^{th} … harmonics of another note, the brain recognizes the octave interval between the two notes. In fact:

Harmonics of C2: C2 **C3** G3 **C4** E4 **G4** Bb4 **C5** D5 **E5** F#5 **G5** Ab5 **Bb5** B5 **C6** …

Harmonics of C3: **C3** **C4** **G4** **C5** **E5** **G5** **Bb5** **C6** …

where the bolded harmonics are coincident.

Thus, when listening to a piece of music, the brain doesn't consider the pitch of each note: instead, it compares the harmonics of two different notes, and from such comparison it gets the interval they form. Few people, however, have what it's called "perfect pitch" or "absolute pitch": when they listen to a frequency, they can recognize the note corresponding to that frequency.

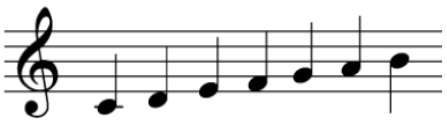
Before the researches of Helmholtz during the nineteenth century (that led to the publishing of his famous text *Sensations of Tone*), the presence of the harmonics hasn't been completely analyzed and comprehended. However, the harmonics have played a primary role in music since before the birth of harmony, when Western music was entirely based upon systems of scales.

A **scale** is a sequences of notes, arranged in ascending or descending order of pitch, that is used as a collection of sounds for making the melodies. In Western music, the scales generally consist of seven notes and their full length is one octave, so that they can be easily extended over different octaves. In particular, the **diatonic scale** is the most famous seven-notes scale. It derives from the old Greek theory: it could be obtained by the adding together two identical four-notes scales, called *tetrachords*, separated by a tone.

The diatonic scale that begins on the note C, also called "C diatonic scale", is:

C, D, E, F, G, A, B, (C)

and its notes correspond to the white keys of the piano. In the figure 1.2 these notes are displaced on the score.



[**Figure 1.2**: The diatonic scale.]

The intervals between E-F and between B-C correspond to a semitone, and the other notes lie a tone distant from the adjacent ones. That is, calling T the tone and S the semitone, the diatonic scale corresponds to the sequence of consecutive intervals T-T-S-T-T-T-S, where the sum of tones and semitones is the amplitude of the octave.

Each note of a scale is called a **degree** of the scale. In the case of the above C diatonic scale, C is the first degree, D is the second, E is the third, and so on. It is therefore a seven-degrees scale.

Although the precise derivation of the diatonic scale from acoustic aspects is matter of discussion (see chapter 5), this scale is strongly related to the perception of the harmonics in the musical sounds: it comprehends all the first 10 harmonics of C but Bb.

By switching the positions of the 5 tones and the 2 semitones of the diatonic scale, many different scales can be produced in the range of one octave. In particular, in the European music before the sixteenth-seventeenth centuries, the scales (that were called **mode**s) were produced by maintaining unaltered the order of the C diatonic scale's tones and semitones, and beginning a new scale from each degree. The result is:

Ionian mode:	C, D, E, F, G, A, B, (C)	(T-T-S-T-T-T-S)
Dorian mode:	D, E, F, G, A, B, C, (D)	(T-S-T-T-T-S-T)
Phrygian mode:	E, F, G, A, B, C, D, (E)	(S-T-T-T-S-T-T)
Lydian mode:	F, G, A, B, C, D, E, (F)	(T-T-T-S-T-T-S)
Mixolydian mode:	G, A, B, C, D, E, F, (G)	(T-T-S-T-T-S-T)
Aeolian mode:	A, B, C, D, E, F, G, (A)	(T-S-T-T-S-T-T)
Hipophrygian mode:	B, C, D, E, F, G, A, (B)	(T-S-T-T-S-T-T)

where the first one (the Ionian mode) coincides with the diatonic scale itself. Although the classification of the modes varies according to the historical period, the above seven scales are known as **diatonic modes**. The interaction between tones and semitones provides a particular mood or feeling to the melodies that are based on each mode.

In all the diatonic modes but the last one (which begins on B) the distance between the first and the fifth degrees is a fifth interval. The acoustic affinity between a couple of notes separated by a fifth is greater than that of any other interval, except the octave. In fact, consider e.g. the C-G fifth interval. The initial harmonics of C are:

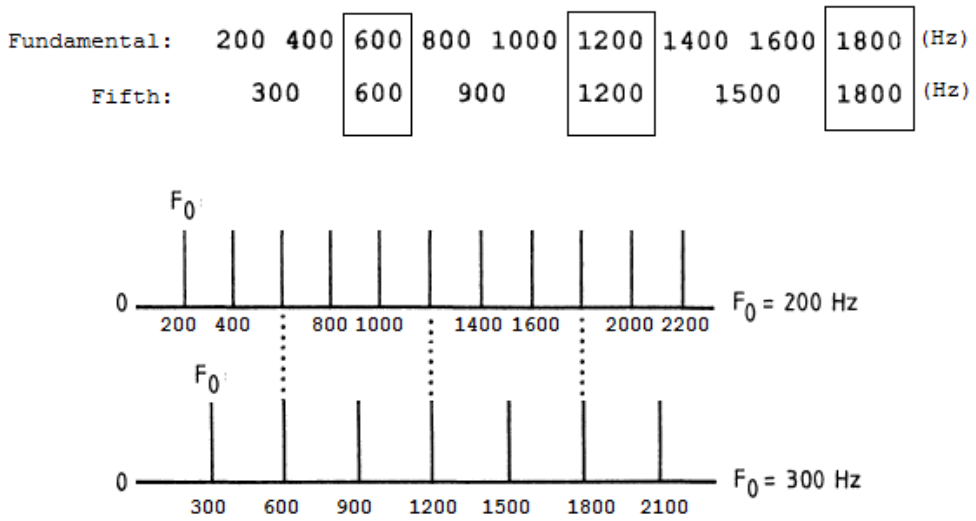
C2, C3, **G3**, C4, E4, **G4**, Bb4, C5, **D5**, E5, F#5, **G5**, Ab5, …

and those of G are:

G2, **G3**, D4, **G4**, B4, **D5**, F5, **G5**, A5, …

Look at the bolded notes: the 2nd harmonic of G coincides with the 3rd harmonic of C, the 4th harmonic of G coincides with the 6th of C, the 6th harmonic of G coincides with the 9th of C, and so on (see also the figure 1.3). Thanks to this (exceptionally) large amount of harmonics that C and G

have in common, the brain easily recognizes the fifth interval when these notes (and every other couple of notes a fifth apart) are listened.



[**Figure 1.3**: Harmonics that two notes a fifth apart have in common.]

2-Brief history of harmony

The medieval scales, also called *Gregorian modes* or *church modes*, have been introduced in the European music by Pope Gregory I (r. 590-604). These scales allowed to collect and organize the body of the liturgical chants in use at the time, and remained the essential theoretical construct until the sixteenth century.

The modes are derived from the old Greek theory, where the seven notes of the diatonic scale could be obtained from the combination of two identical 4-notes scales (tetrachords). In fact, the diatonic scale can be produced by adding together two tetrachords corresponding to the sequence T-T-S of tones (T) and semitones (S), separated by a tone placed in the middle: T-T-S-(T)-T-T-S. As discussed in the previous chapter, the modes differ one to each other because they start on different diatonic scale's notes, so that the two semitone intervals are placed in different positions.

In the first centuries after the introduction of the Gregorian modes, the Western musical practice have mostly neglected the possibility of superposing melodies. Among the first available examples of polyphonic music, the book *Enchiridion Musicæ*, written by the Flemish monk Hucbald (840?-930-32?), describes two melodies proceeding in parallel and separated by a fourth, a fifth or an octave interval. This technique was called *parallel organum*.

A century later, the Italian monk and theorist Guido d'Arezzo (990-1050) synthesizes many elements of various notational practices in his *Micrologus* and *Antiphonarium*. He introduces the modern staff notation, that replaced the neumatic notation. Behind the parallel organum, he includes oblique (diverging or converging) motion and contrary motion of voices.

Until the end of the 11th century, the note-against-note technique (i.e., two synchronized melodies that proceed with repetitive rhythm) was the only polyphonic practice to be largely adopted. This composing style, where simultaneous notes create a sequence of equally time-spaced intervals, is called *homophony*. With the adventure of 12th century, it appears the usage of superposing a faster melody against a slower bass.

In middle of the thirteen century, the theorist Franco of Cologne establishes the first classification of the consonant intervals, distinguishing between "perfect consonances" (the octave only), "middle consonances" (fourths and fifths), and "imperfect consonances" (major and minor thirds). He consider the sixths dissonant intervals, but declares that their dissonance is less strong than that of augmented fourths, augmented fifths, and sevenths. Marchetto of Padua, who also lived in the 13th century, specifies that the dissonant intervals need special treatment: these intervals should be followed by consonant ones, a practice that in the following centuries will be known as "resolution of the dissonance". Moreover, he classes the fourth among the dissonant intervals (this is not a wrong statement, as it will be discussed in chapter 4).

Among the most relevant contributions of the fourteenth century, in France and Italy it is remarkable the work of Philippe de Vitry, Guillaume de Machaut, Jacopo da Bologna, Francesco

Landini, Jean de Muris, Ghirardello da Firenze and others. They are exponents of the *Ars Nova* ("new art") musical style, characterized by the increased use of smaller note values and new rhythmic solutions. The designation "Ars Nova", to be specific, was introduced around 1320 by de Vitry as the title of its four-manuscripts treatise. This development was in part anticipated by Pierre de la Croix in the second half of the thirteen century.

By the fifteenth century, under the influence of Britannic composers such as John Dunstable, the third and the sixth intervals become more largely adopted in the European music. In this period, Johannes Tinctoris (1436-1511) writes one of the earliest dictionaries of musical terms, entitled *Terminorum musicae diffinitorium*.

The evolution of harmony from the system of church modes encompasses several centuries and various styles of music. Until the sixteenth century, only the horizontal components of the music was taken into consideration: the chords were only an accidental result of the cohesion of independent voices. During the Renaissance (about 1450-1600), a gradual growth of the perception of harmonic relations and the vertical dimension of simultaneously sounded melodies toke place. This evolution led to the synthesis from which the tonal system takes its origins.

Whereas the ecclesiastical environment continued to sustain the production of music, during the Renaissance the musical activities gradually shifted to the courts, where the finest composers wrote both secular pieces and sacred works for the court chapel. Both in the Middle Ages and the Renaissance, vocal music was more important than instrumental music, and choirs usually sung *a cappella*, i.e. without instrumental accompaniment. During the sixteenth century, however, instrumental music became less subordinate to the vocal practice, and more works specifically for instruments began to appear.

Many of the most relevant Renaissance musicians came from Italy, the Flanders (which today includes parts of the Netherlands, Belgium, and France), Germany, England, and Spain. Among them, there were Josquin Desprez (1440-1521), Thomas Tallis (1510-1585), Giovanni Pierluigi da Palestrina (1525-1594), Roland de Lassus (1532-1594), Giaches de Wert (1535-1596), William Byrd (1543-1623), Tomás Luis de Victoria (1548-1611), Luzzasco Luzzaschi (1545-1607), Luca Marenzio (1553-1599), Giovanni Gabrieli (1557-1612), John Dowland (1563-1626), Carlo Gesualdo (1566-1613), Thomas Campion 1567-1620), and Claudio Monteverdi (1567-1643), whose innovative work is largely held responsible for the change from the Renaissance to the Baroque periods.

Since the beginning of the Renaissance, various theorists have contributed to the analysis and the description of what was happening in Western music. Important treatises are those of Gioseffo Zarlino (author of *Istitutione harmoniche*, 1558), Johann Lippius (*Synopsis musicae novae*, 1612), Adriano Banchieri (*Cartella musicale*, 1614), Michael Praetorius (*Syntagma Musicum*, 1619), Lorenzo Penna (*Li primi albori musicali*, 1672), Giovanni Bononcini (*Musico Prattico*, 1673), Angelo Berardi (*Il perchè musicale*, 1693), Francesco Gasparini (*L'armonico pratico al cimbalo*, 1708), Antonio Bruschi (*Regole per il contrapunto e per l'accompagnatura del basso continuo*, 1711), and François Campion (*Traité d'accompagnement et de composition, selon la regle des octaves*, 1716).

Finally, it came the monumental *Traitè de L'armonie* (1722) by Jean-Philippe Rameau, one of the most important treatises in the history of music. Although after Rameau many improvements have been made, his work fully comprises the foundations of modern harmony.

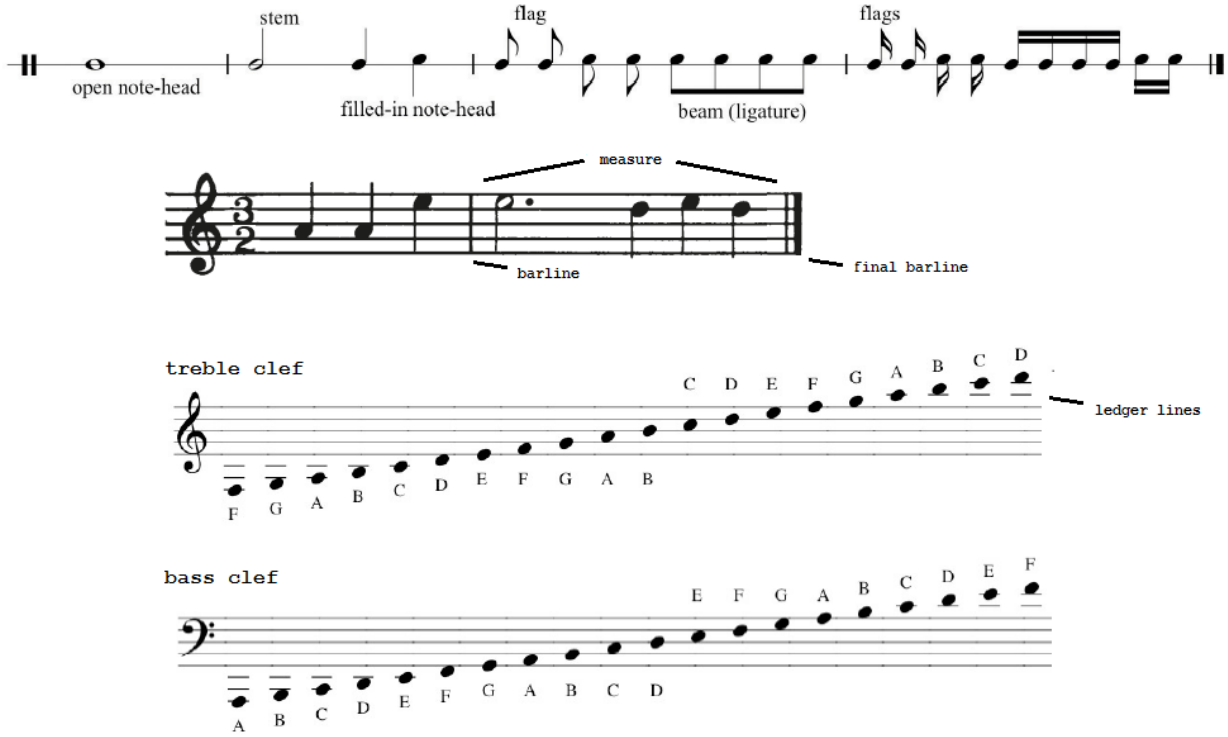
With the establishment of the principles of harmony in the theoretical background of the European music, a new and deeper awareness of the sound phenomena led the composers to the creation of artistic works of great complexity and perfection. Too many wonderful examples have been produced by the great masters of the common practice period. There is no need to list them.

3-Music fundamentals and notation

Although many types of music notation have been introduced through the history of written music, the so-called **common notation** is currently the most widespread format for writing music. Since the composition of scores is also related to the technical aspects characterizing each musical instrument (or voice), the symbols and formats listed below are subjected to a great variety of additions that cannot be described in only one chapter.

Notes are associated to both a pitch and a time duration. They are written on the **staff** (plural=*staves*), where the time flowing proceeds horizontally (going from the left to the right) and the pitch is related to the vertical dimension. The 5 horizontal parallel lines composing the staff are the reference heights for the pitches: higher-pitched notes are located higher on the staff. If many staves have to be played at the same time, they are connected together by a long vertical line at the left hand side.

Notes are indicated by drawing little ovals, called **notehead**s, which can be empty or filled-in. A **stem** is eventually attached to them, and one or more **flag**s on the top of the stem can be present. The noteheads can be placed both on the lines and in between two lines; for extending the staff higher and lower, small lines called **ledger lines** are to be used. See figure 3.1.



[**Figure 3.1**: Notes' symbols and names, ledger lines, measures, final barline.]

A **melody** (also called *melodic line*, musical phrase, or *theme*) is a sequence of notes that has some aesthetic purpose. The rhythm of a melody is the placement of the notes in time, which is obtained by assigning a **rhythmic value** or **note value** to the notes: the rhythmic value indicates the notes' relative durations with respect to the other notes. The actual time-extent of the notes depends on the speed at which the song is played. The presence of empty or filled-in noteheads, flags and stems allows to identify the different rhythmic values that the notes can assume.

The biggest rhythmic value is called **double whole-note** or **breve**. The other rhythmic values are submultiples of the breve: to be specific, they correspond to one breve divided by the powers of two, that is, a **whole-note** is half the length of a breve, a **half-note** is half the length of a whole-note, a **quarter-note** is half the length of a half-note (i.e., whole-note/4), an **eight-note** is half the length of a quarter-note (whole-note/8), a **sixteenth-note** is half the length of a eight-note (whole-note/16), and so on.

Notes will be drawn with the stem if their rhythmic value is smaller than the whole-note, and they are filled-in ovals if their rhythmic value is smaller than the half-note; whenever the notes are smaller than the quarter-note, the stem can have one or more flags (one flag the eight-note, two flags the sixteenth-note, and so on). See the figure 3.2.

The stems (along with their flags) can be equally drawn either up or down: in the first case the stem is attached to the right side of the oval, in the second case to the left. One can often see the note's stems being connected together by (roughly horizontal) lines, called **beam**s, that join the note's flags perpendicularly to the stems. Beams are only used to make the rhythmic subdivision easier to read.

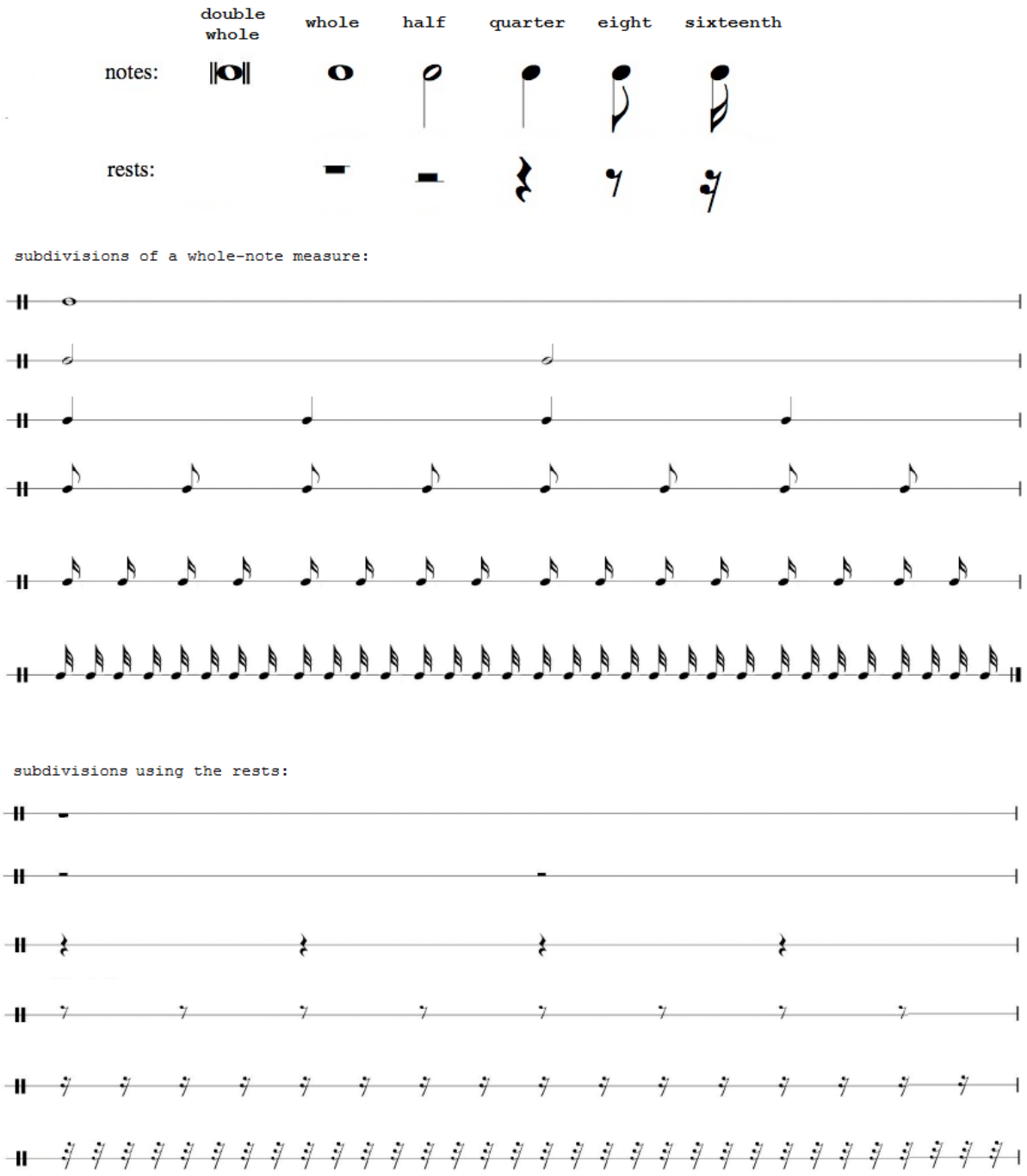
A pause (i.e., the interruption of a melody) is called a **rest**. Rests' rhythmic values are catalogued in the same way as the notes (as shown in figure 3.2).

For to assign the diatonic scale's notes C, D, E, F, G, A, B to the staff lines, the **treble clef** and the **bass clef** symbols are the most commonly used ones. They are written at the beginning of each staff's row. The bottom line of the staff is identified as the 1^{st} line, and the treble clef is placed on the second line, which becomes G. Then, the third line will become B and the space in between the second and the third lines is A (see figure 3.1), and so on. The bass clef is written on the fourth line, which becomes F (and the same criterion applies to the other notes). Although these clefs can be placed on different lines, it is rare to find them anywhere but in these positions.

The staff is divided into short sections called **measure**s or **bar**s separated by vertical lines called **barline**s. On the composition's ending the **final barline** symbol is written. The sum of the rhythmic values of the notes and the rests laying in any measure must correspond to the measure's rhythmic value.

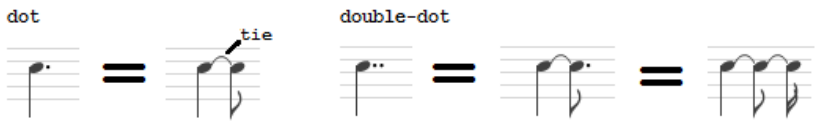
For joining the notes together to form a continuous sound a curved line called **tie** is used (see figure 3.3). One or more notes of any length may be tied together; the rhythmic value the tied

notes stand for will be the rhythmic value of all the notes added together. Tied notes can be used e.g. for sustaining the note while going from one measure to the following.



[**Figure 3.2**: Rhythmic values of notes and rests; subdivisions of a whole-note measure.]

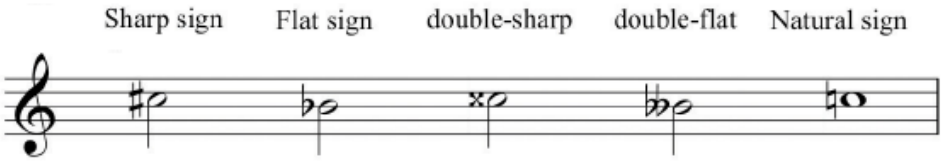
If some note has to assume one time and a half its rhythmic value, a **dot** right after the notehead can be added. For incrementing the rhythmic value by 3/4 of its value, the **double-dot** is used: the double-dot symbol, therefore, is equivalent to putting a dot to the increment provided by the first dot. Observe that a dotted note is the same as a note and a half-valued note tied together.



[**Figure 3.3**: Dot, double-dot, tie.]

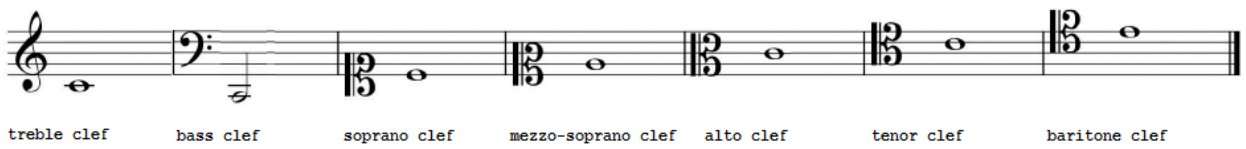
The **sharp** (#) and the **flat** (b) symbols, which are placed right before the noteheads, indicate that the note is to be considered respectively a semitone higher or a semitone lower. There also exist symbols for the whole tone-shifting: the **double-sharp** (similar to an "X") transports the note a tone higher, and the **double-flat** (bb) a tone lower. These four semitone-shifts and tone-shifts are called **accidentals** and are shown in figure 3.4. If an accidental is placed upon a certain note, then, inside the measure where the note is found, all the following manifestations of such note are affected by the accidental. This happens unless another special symbol is placed: the **natural symbol**, which cancels the accidentals being written on its own line (or space) and measure before it. There is no need to pitch-back the raised/lowered notes when passing from a measure to the following one because the barlines don't keep alive both the accidentals and the natural signs.

Some special accidentals are placed at the beginning of each staff's row, instead of being found inside the measures. They are positioned right after the clef symbol: they are "active" through all the row, and have influence on the notes of all the octaves of the line/space where they are written. The natural sign cancels them only in the measure and the octave it is placed. The list of these special accidentals is called **key signature**. The key signature can change during the piece; as will be discussed in chapter 6, it is related to the choice of the tonality, and it is composed either by flats only or by sharps only (their quantity depends on the tonality).



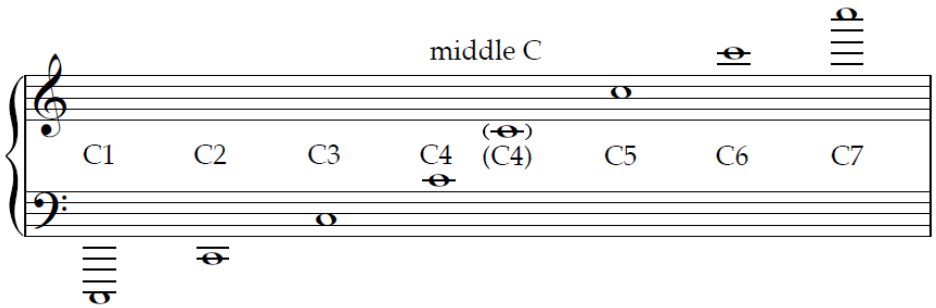
[**Figure 3.4**: Treble clef with accidentals and the natural symbol.]

The set of pitches, from lowest to highest, that an instrument can sound is called **range** of the instrument, and a specific part of the range is called **register** (e.g., the range can be divided in bass register and high register). Since instruments and voices have different ranges and registers, another clef symbol exists, the **C clef**, which is moveable and makes its staff line to be C. The C clef is called differently depending on the line upon which is placed, and its names coincide with the vocal registers: **baritone clef** (fifth line), **tenor clef** (fourth line), **alto clef** (third line), **mezzo soprano clef** (second line) and **soprano clef** (first line). Instruments with high ranges usually adopt the treble clef, instruments with low ranges usually adopt the bass clef, and transposing instruments (for which the notes are written higher or lower than where they sound) usually adopt the C clef. The list of the clefs is given in figure 3.5.



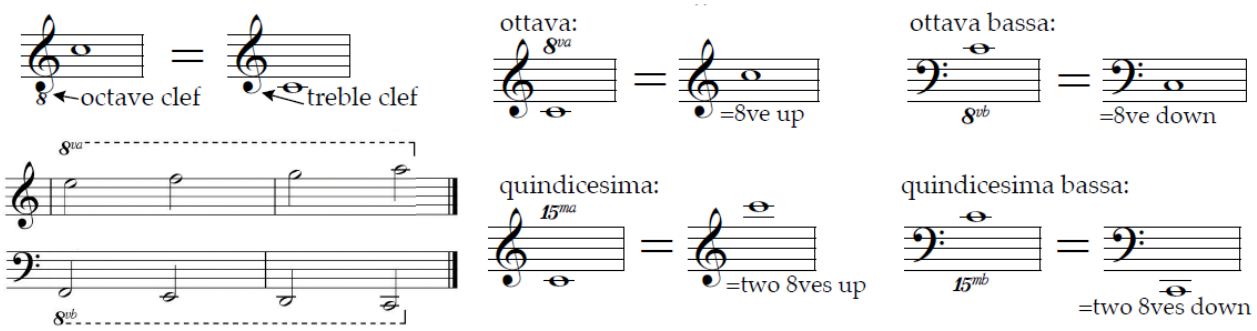
[**Figure 3.5**: Treble clef, bass clef, C clefs.]

In the case e.g. of scores for piano or for choir, the **grand staff** is used: it is composed by both the treble clef and the bass clef, and the notes can span through the two staves. The middle C4 is notated differently depending on whether it belongs to the treble or bass clef (see figure 3.6).



[**Figure 3.6**: Grand staff with middle C4.]

Among the pitch-related symbols, there exist various styles to indicate whenever the notes are to be considered one (or more) octave(s) above or below where they are written (see figure 3.7):

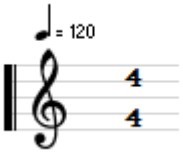


[**Figure 3.7**: Ottava, Octave clef, Quindicesima.]

In Western music, time is organized by dividing it into a sequence of regular periods, called **beat**s. The music events usually tend to happen right at the beginning of a beat: the emphasis that is given to the subdivision into beats give the music a regular and recognizable motion. The rhythmic values are written in relation to the rhythmic value of the beats, which is called the **beat unit**: if the quarter-note receives e.g. one beat, then the half-note will receive two beats, and the whole-note four beats.

The **tempo** is the speed of the beats, i.e. their time duration, and it is given by the metronome marking at the beginning of the score (and, eventually, at any spot where it changes). The

metronome marking indicates the number of **BPM** (Beats Per Minute), i.e. the number of beats that are present within one minute. E.g., if one writes that a quarter-note (the beat unit) is equal to 60 BPM, then there are 60 quarter-notes in one minute, so that each quarter-note corresponds to 1 second. Thus, the greater is the number of BPM the faster is the song.



[**Figure 3.8**: Example of metronome marking.]

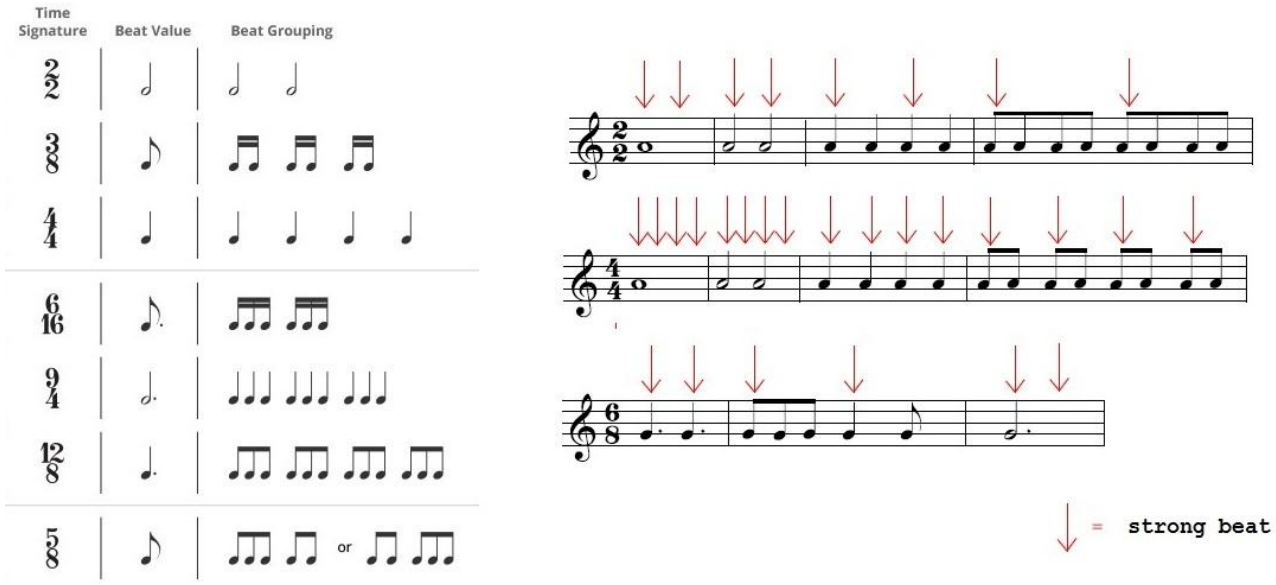
If a metronome marking is not available, the tempo can also be specified by writing some Italian terms above the notes. From slowest to fastest, some of them are the following: *Grave* (solemn), *Largo* (very slow), *Lento* (very slow), *Adagio* (slow), *Andante* (moderately slow), *Moderato* (moderate), *Allegro* (fast), *Vivace* or *Vivo* (lively), *Presto* (very fast), and *Prestissimo* (very fast). There also exist terms for marking a gradual changes of the speed, such as *accelerando* (abbreviated *accel.*, accelerating, getting faster), *ritardando* (abbrev. *rit.*, slowing down) and *rallentando* (abbrev. *rall.*, gradually slower).

The beats lying in each measure are divided into groups, usually including two, three or four beats, so that a repetitive pattern is produced if many consecutive measures have the same group subdivision. The most important beat of any measure is the first one, which is called the **downbeat**. The downbeat and the first beat of any group are called **strong beat**s, while the others are the **weak beat**s. The notes that are synchronized with the strong beats have to be emphasized by the executor, so that the beat subdivision into measures and groups becomes explicit. The **time signature** or **meter signature** indicates how the beats of a measure are grouped, that is, it specifies the way the strong and the weak beats are displaced.

The time signature is represented by a "fraction", where the upper value is the number of beats which are present in the measure, and the bottom number is the beats' rhythmic value (the beat unit). The upper number also establishes the way the beats are grouped:

- If the upper number is a multiple of 2 and not a multiple of 4 (i.e., usually 2 and 6), then the meter signature is a **duple meter**, and the measure is divided into 2 equal groups of beats.
- If the upper number is a multiple of 4 (usually 4, 8 or 12), then the meter signature is a **quadruple meter**, and the measure is divided into 4 equal groups of beats.
- If the upper number is a multiple of 3 and not a multiple of 2 or 4 (usually 3 and 9), then the meter signature is a **triple meter**, and the measure is divided into 3 equal groups of beats.
- All the other cases are called **asymmetric time signatures**: the measure includes a combination of different groups.

Thus, duple meters have 2 groups, triple meters have 3 groups, and quadruple meters have 4 groups. The first beat of each group is a strong beat, the others are weak beats. E.g., the 3/2 time signature (a triple meter) corresponds to 3 groups of 1 half-note beat, the 8/4 time signature (quadruple meter) corresponds to 4 groups of 2 quarter-note beats, the 6/8 time signature (duple meter) corresponds to 2 groups of 3 eight-notes, and the 3/4 time signature (triple meter, the waltzes' rhythm) involves 3 groups of 1 quarter-note beat. Common examples of asymmetric time signatures are 5/8, which is decomposed into a group of 3 beats and a group of 2 beats, and 7/8, whose group-subdivision can be 2+2+3, 3+4, 2+3+2, ….



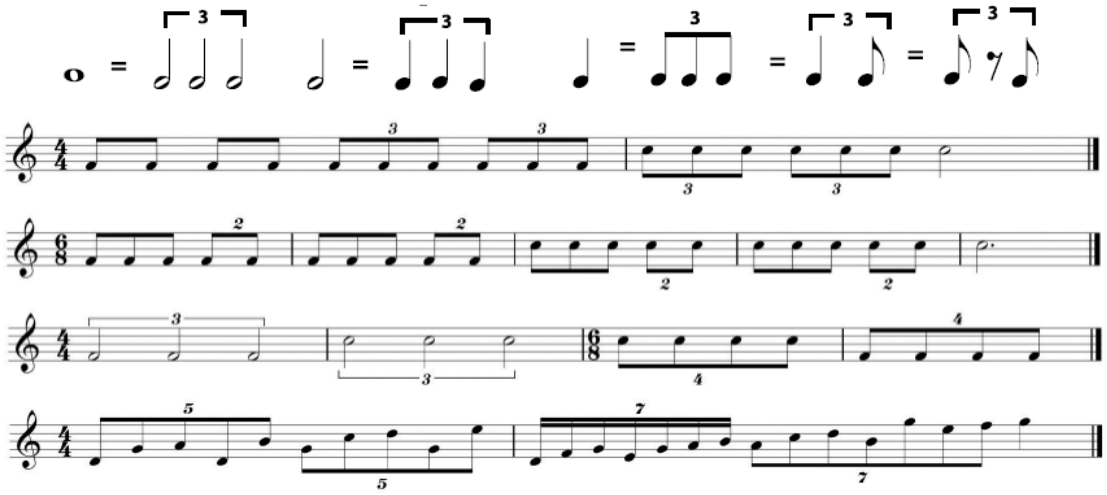
[**Figure 3.9**: Examples of time signatures, beats grouping in different meters, strong beats.]

The 2/2 and 4/4 time signatures, respectively called **cut time** and **common time**, are of very common usage. They are often written by drawing respectively the symbols of a vertically barred "C" and a bare "C" in place of the respective fractions.

Different combinations of tempo and meter signature can provide the same setting for the composer: e.g., assigning a quarter note to 60 BPM and adopting the 3/4 time signature is the same as assigning a quarter note to 120 BPM and adopting the 3/2 time signature. Normally, the time signature that makes the music easier to read is adopted. The meter can change during the song; in such cases, the new meter signature will be written at the beginning of the first measure after the change (see figure 3.10). There also exist compositions, such as the ancient Gregorian chants and some non-Western music, where no meter can be identified.

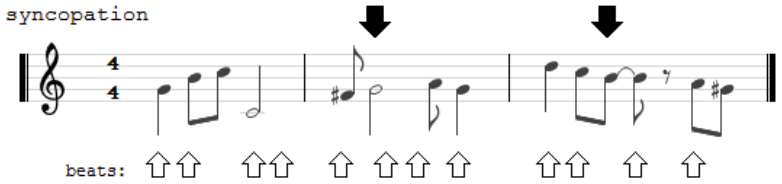
A **borrowed division** occurs whenever the rhythm of a piece involves some beats that belong to a different meter. Such time subdivisions can be obtained by using the **tuplet**s or **grouplet**s symbols. Among them, the **triplet**s are the most used. Triplets divide a rhythmic value into three equal portions (rather than into powers of two). In practice, the rhythmic value that has to be divided is considered as it was dotted, so that it contains three halves of itself: the notes that fill this virtually expanded time-length are grouped together by a bracket and/or a beam that is placed

above them (see figure 3.10). The number "3" is then written above the group. The **duplets** and **quadruplets** make the opposite operation of creating respectively 2 and 4 equal rhythmic values into a group of three beats. There also exists **quintuplets**, **sextuplets** and **septuplets** too, where the same criterion applies.



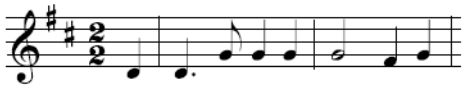
[**Figure 3.10**: Examples of borrowed divisions.]

The **syncopation** or **syncopated rhythm**, one of the most customary features of Western music, occurs if an arrangement of notes places accent on a weak beat (or a weak element of the beat's subdivision), instead of being synchronized with the strong beat. In other words, the rhythm puts an emphasis when it is unexpected. E.g., a "long" note that is placed on a weak beat produces the effect of syncopation. See e.g. the figure 3.11.



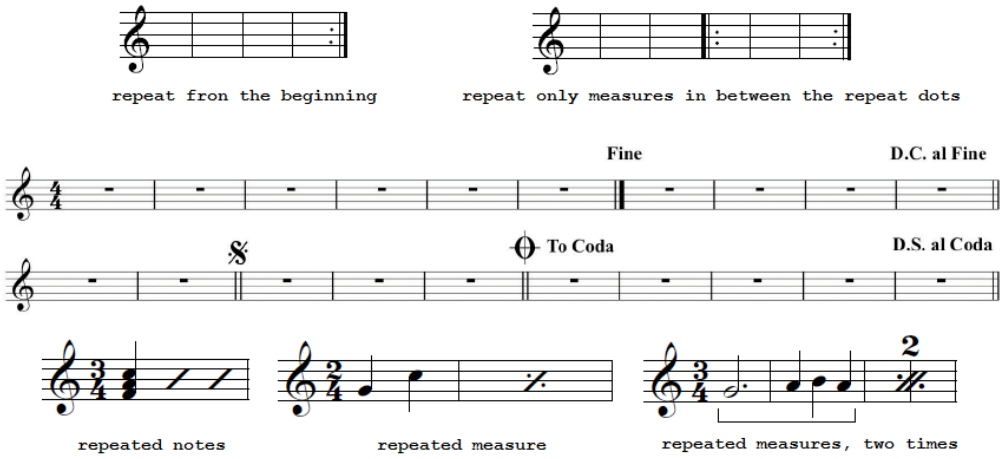
[**Figure 3.11**: Examples of syncopation.]

Sometimes happens that the composition begins before the first barline, in the middle or at the end of a shorter measure placed right before the first measure. This usage is called **anacrusis** or **pickup measure** (see figure 3.12).



[**Figure 3.12**: Anacrusis.]

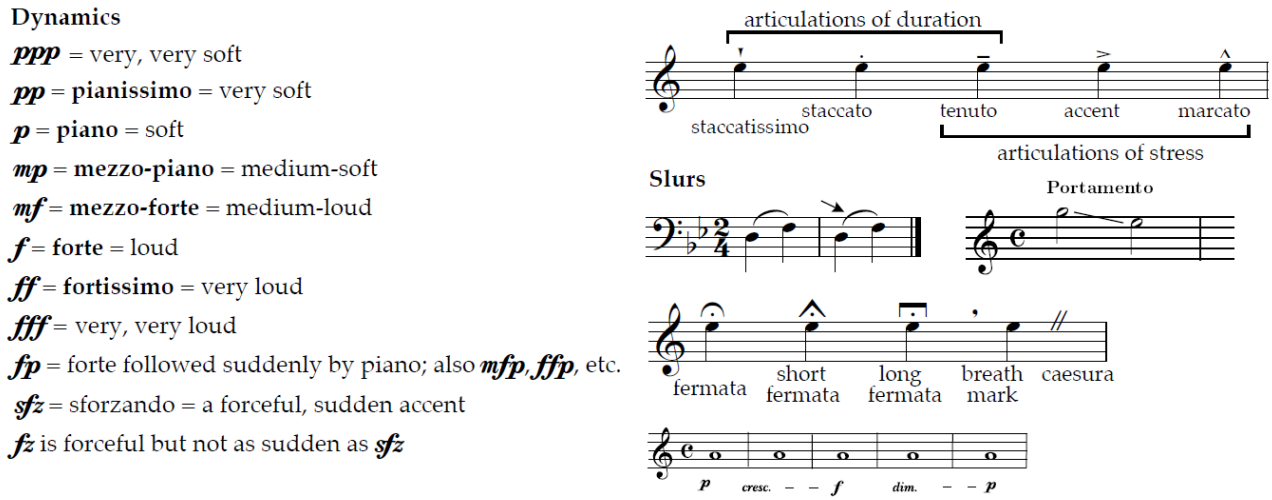
Whenever a measure or a group of subsequent measures has to repeated, the **repeat dots** are placed at the beginning and at the end of the involved section, respectively behind the first and the last barlines. If no beginning repeat dots are present, the performer has to go all the way back to the beginning of the score and repeat from there. There are some tricks to denote the repetition of sections: e.g., to write *D.C.* and *D.C al fine*, or *D.S.* and *D.S. al coda*. "D.C." means repeat back to the beginning; "D.C. al fine" tells the performer to end the piece where the *fine* marking in placed. "D.S." means repeat back to a special symbol (see figure 3.13); "D.S. al coda" tells the performer to repeat back to the symbol until the *to coda* marking, then "jump" to a separate final section. Moreover, it is possible to explicitly write a number of measures that correspond to the duration of the repeated section, but without filling them with the notes: in place of the notes, special symbols are placed, as shown in the figure 3.13.



[**Figure 3.13**: Commonly used repeat signs.]

A variety of signs that indicate how to play one or more notes also exists. The **dynamics** marks (see figure 3.14) indicate the relative loudness of the notes. The *crescendo* vs. *diminuendo* (abbreviated *cresc*. and *dim*.) markings indicates respectively to increase or to decrease the loudness while playing group of notes.

The **articulations** are used to specify how the duration or the stress of a group of notes should be performed; they strongly depend on the particular instrument that is making the music. Among the most commonly used, *staccato* indicates that the notes are to be sounded shorter than their own rhythmic value (and *staccatissimo* means "more staccato"); *legato* or *tenuto* is the opposite of staccato (the notes are very connected). An **accent** requires that the notes stand out more than the other notes around them, and *marcato* means "marked": the notes are enough accentuated and/or spaced to be better perceived. The **slurs** are signs for connecting different pitches (in a way that depends on the instrument that is making the music), and the **portamento** is a slide between the two notes that includes all the pitches in between. The **fermata**s signs indicate to keep the note until the conductor or soloist do something.



[**Figure 3.14**: Dynamics, articulations, slurs, portamento, fermata, crescendo vs diminuendo markings.]

4-Intervals and scales

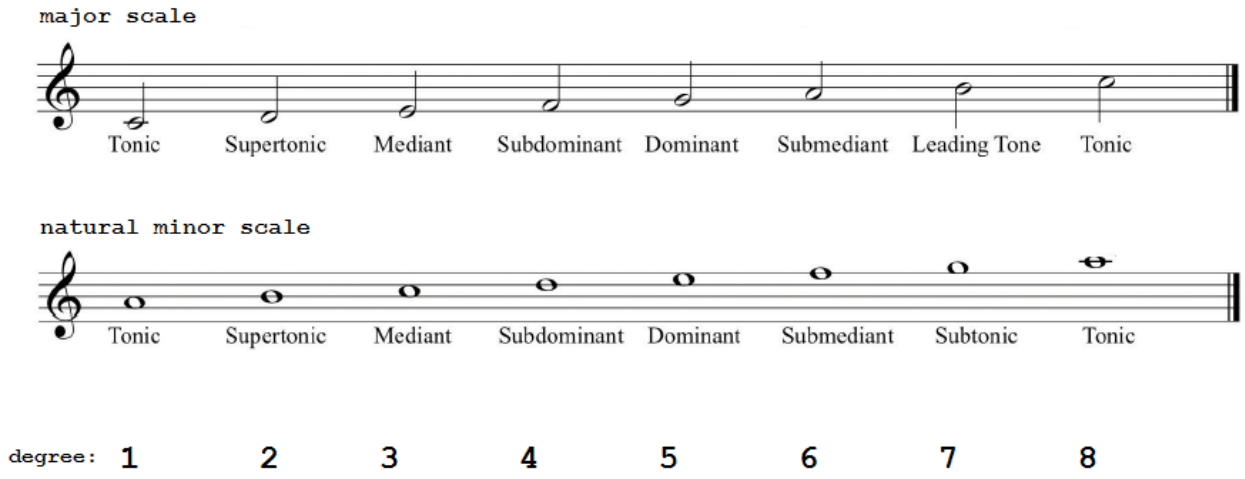
The diatonic scale beginning on C has the form:

C, D, E, F, G, A, B, (C)

The diatonic scale coincides with the **major scale**. The same seven notes are involved in the **natural minor scale**:

A, B, C, D, E, F, G, (A)

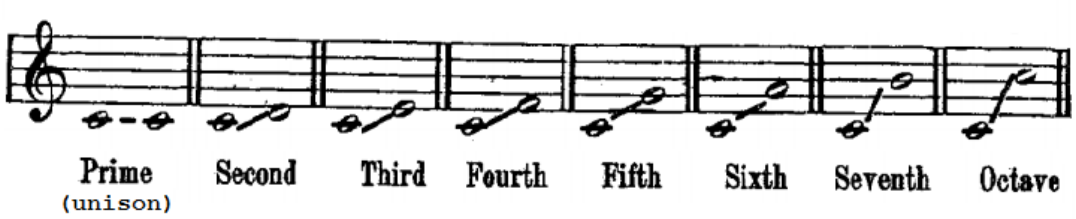
The first degree of these scales, respectively C and A, is called **tonic** or **fundamental tone**. The second degree is called **supertonic**, the third **mediant**, the fourth **subdominant**, the fifth **dominant** and the sixth **submediant**. The seventh degree of the natural minor scale is called **subtonic**, and the seventh degree of the major scale is the **leading-tone**.



[**Figure 4.1**: C major scale and A natural minor scale.]

The mediant, the submediant and the subtonic degrees of the natural minor scale lie one semitone lower than the mediant, the submediant and the leading-tone of the major scale. The other degrees are coincident, i.e. they form the same intervals with their tonic. In particular, the amplitude of the tonic-mediant interval distinguishes a major scale from a minor scale: whenever it corresponds to 4 semitones, as in the major scale, the scale is a major or "major-like"; if it corresponds to 3 semitones, as in the natural minor scale, then the scale is a minor or "minor-like".

The intervals between the diatonic scale's notes are called *prime*, *second*, *third*, *fourth*, *fifth*, *sixth*, and *seventh* depending on the number of degrees they occupy, including both the notes forming the interval. E.g., the fourth interval D-G occupies 4 degrees: D, E, F, G. The prime interval is the interval between a degree and itself. The intervals between the tonic and the other notes of the diatonic/major scale are shown in figure 4.2.



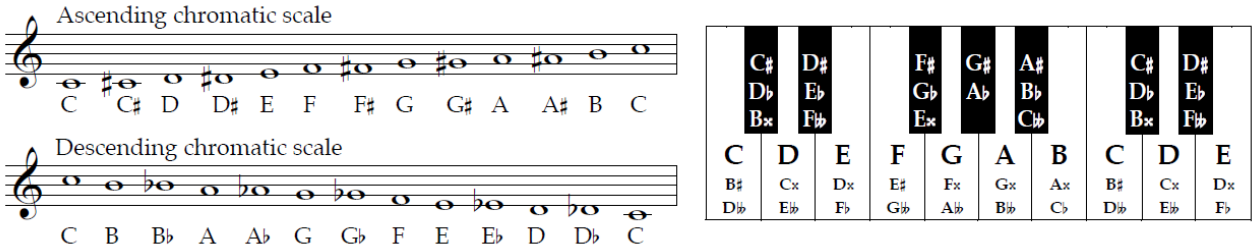
[**Figure 4.2**: Intervals between the diatonic/major scale's degrees and the tonic.]

Observe that by moving a diatonic degree a fifth higher or a fourth lower produces the same note in different octaves (e.g., G1 and G2 are respectively a fourth below and a fifth above C2), and the same applies to the other couples of intervals: third-sixth, second-seventh, prime-octave.

However, the names "prime", "second", "third", … are not sufficient for completely characterizing the intervals: e.g., both F-A and E-G are third intervals, since they involve three degrees, but they correspond to a different number of semitones. Therefore, the subdivision of the octave in 12 semitones is needed for properly classifying the intervals. The scale that is produced from the equal-tempered subdivision of the octave in 12 half-steps is called **chromatic scale**:

C, C#/Db, **D**, D#/Eb, **E**, **F**, F#/Gb, **G**, G#/Ab , **A** , A#/Bb, **B**, (**C**)

where the bolded letters mark the diatonic scale, and the non-diatonic notes can be written either using a sharp or a flat. To be specific, as will be discussed later, the ascending form of the scale is written with sharps, and the descending with flats, as shown in figure 4.3.



[**Figure 4.3**: The chromatic scale.]

The chromatic scale allows to transport the intervals higher or lower without changing the number of semitones they comprehend. E.g., the third interval F-A can be written one semitone higher: F#-A#, or Gb-Bb. In other words, the chromatic scale permits the **transposition**: a group of notes (melodies or chords) is said to be *transposed* when all the notes are written the same interval higher or lower than their original pitch. In particular, the chromatic scale allows to write the diatonic scale at different pitches: after the transposition, some notes will have acquired or lost accidentals for preserving the nature of the diatonic scale.

The presence of 12 notes in the chromatic scale produces 12 intervals (in the range of the octave):

- **unison** (or *prime*) [0 ¢]: the distance between a note and itself.

- **minor second** [100 ¢]: the semitone.
- **major second** [200 ¢]: the tone. E.g., the tonic-supertonic interval in both the major and the natural minor scales.
- **minor third** [300 ¢]: e.g., the tonic-mediant interval in the natural minor scale.
- **major third** [400 ¢]: e.g., the tonic-mediant interval in the major scale.
- **perfect fourth** [500 ¢]: e.g., the tonic-subdominant interval in both the major and the natural minor scales.
- **tritone** [600 ¢]: the interval between the subdominant and the leading-tone in the major scale, or the supertonic-submediant interval in the natural minor scale.
- **perfect fifth** [700 ¢]: e.g., the tonic-dominant interval in both the major and the natural minor scales.
- **minor sixth** [800 ¢]: e.g., the tonic-submediant interval in the natural minor scale.
- **major sixth** [900 ¢]: e.g., the tonic-submediant interval in the major scale.
- **minor seventh** [1000 ¢]: e.g., the tonic-subtonic interval in the natural minor scale.
- **major seventh** [1100 ¢]: e.g., the tonic-leading-tone interval in the major scale.
- **octave** [1200 ¢]: the total length of the scale.

The placing of an accidental against a note of the C diatonic scale produces a **chromatic alteration**. If a sharp is used, the note is said to be **chromatically raised**, if a flat is used, the note is **chromatically lowered**. In both the cases, the note is said to be **chromatically altered**.

By chromatically altering either the upper or the lower note of the above intervals, the following happens:

- If a minor interval is enlarged by chromatically altering one of its notes (i.e., the upper note is chromatically raised, or the lower note is chromatically lowered), then it becomes a major interval. E.g., the minor third E-G becomes either the E-G# or the Eb-G major third.
- If a major interval is reduced by chromatically altering one of its notes, then it becomes a minor interval. E.g., the major sixth C-A becomes either the C#-A or the C-Ab minor sixth.
- If a perfect or a major interval is enlarged by chromatically altering one of its notes, then it is called **augmented interval**. E.g., the perfect fifth D-A becomes either the D-A# or the Db-A augmented fifth; the major third G-B becomes either the G-B# or the Gb-B augmented third.
- If a perfect or a minor interval is reduced by chromatically altering one of its notes, then it is called **diminished interval**. E.g., the perfect fifth D-A becomes either the D#-A or the D-Ab diminished fifth; the minor third A-C becomes either the A#-C or the A-Cb diminished third.

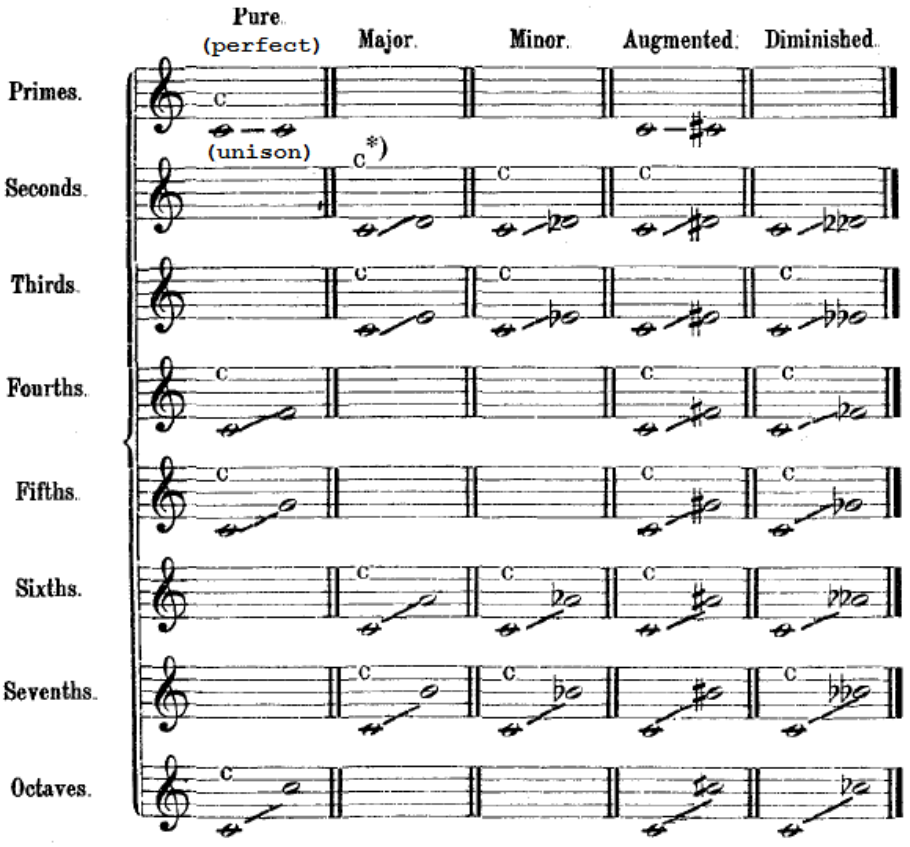
Every interval that includes a chromatically altered note preserves the name (prime, second, third, …) of the interval without the chromatic alteration. E.g., the interval G-Bb is a minor third, because

the involved diatonic degrees are G, A and B; but the interval G-A#, which has the same amplitude of G-Bb, is an augmented second, because the diatonic degrees are only G and A.

The same applies to the intervals where both the upper and the lower notes have been chromatically altered: in such a case, a minor interval can become augmented, and a major interval can become diminished. E.g., transforming the minor sixth A-F into Ab-F# produces an augmented sixth; transforming the major third F-A into F#-Ab produces a diminished third.

The tritone between F and B is the only interval that doesn't need accidentals for being either an augmented fourth (F-B) or a diminished fifth (B-F). Observe that F and B are the only notes of the C diatonic scale which are separated by a tritone interval.

In the figure 4.4 the full list of the intervals is shown.



[**Figure 4.4**: All the intervals in the range of the octave.]

It is also to be noted that the augmented and diminished intervals can have the same amplitude of a major, minor or perfect interval: the labels "augmented" and "diminished" only indicate that an interval has been produced by a chromatic alteration (except the special case of the tritones F-B and B-F).

The addition of an octave to the above listed intervals produces the ninth interval (second+octave), the tenth interval (third+octave), the eleventh interval (fourth+octave), and so on. These intervals

are very similar to the original ones, since the octave-extension doesn't change their main acoustic qualities. Also their names continue to be major/minor, augmented/diminished or perfect: e.g., C2-E3 is a major tenth, C2-G3 a perfect twelfth, C2-F#3 an augmented eleventh.

Transporting the lower note of an interval to the octave above, as well as transporting the higher note to the octave below, produces an **inversion** of the interval. That is, an inverted interval has its notes interchanged: e.g., G-C is the inversion of C-G. The inversions of all the intervals but the tritone (which divides the octave in two equal parts) have different amplitudes than the original interval. To be specific:

- The inversion of the unison is the octave.
- The inversion of the minor second is the major seventh.
- The inversion of the major second is the minor seventh.
- The inversion of the minor third is the major sixth.
- The inversion of the major third is the minor sixth.
- The inversion of the perfect fourth is the perfect fifth.
- The tritone remains the tritone. To be specific, the inversion of the augmented fourth is the diminished fifth, and the inversion of the diminished fifth is the augmented fourth.
- The inversion of the perfect fifth is the perfect fourth.
- The inversion of the minor sixth is the major third.
- The inversion of the major sixth is the minor third.
- The inversion of the minor seventh is the major second.
- The inversion of the major seventh is the minor second.
- The inversion of the octave is the unison.

Observe that the inverted major intervals are minor and the inverted minor intervals are major; the inverted diminished intervals are augmented and the inverted augmented intervals are diminished; the inverted perfect intervals remain perfect.

Couples of notes that have some of their initial harmonics in common often form a **consonant interval**. To be specific, in the range of the octave the consonant intervals are:

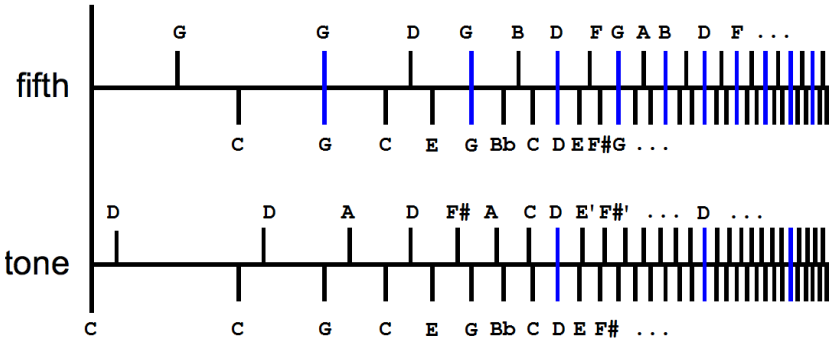
- Octave and unison
- Perfect fifth
- Perfect fourth (not always)
- Major and minor thirds
- Major and minor sixths

The remaining ones, the tritone, the (major and minor) seconds, and the (major and minor) sevenths are **dissonant interval**s.

By looking at the list of the inversions of the intervals, it can be noted that the inversions of the consonant intervals are consonant, and the inversions of the dissonant intervals are dissonant.

As discussed in chapter 1, the human brain recognizes the intervals by comparing the harmonics of the two involved notes. The consonance or dissonance of an interval is related to the amount of computation that is required by the brain for its identification: as the computation becomes easier, the consonance of the interval becomes greater, and vice-versa. E.g., the octave is the easiest interval to be recognized because the two notes have exactly the same harmonics, and therefore it is the most consonant interval.

The term "dissonant", anyway, it's not related to a bad or fastidious sound: on the contrary, for an interval or a chord to be dissonant means exclusively that it creates tension, that is, it generates in the listener the need for another interval or chord to follow. From an acoustic point of view, the sound of the dissonant intervals has inequalities called "beats". In chapter 9 the acoustic aspects of the consonance or dissonance of the intervals will be discussed.



[**Figure 4.5**: Harmonics of two notes a perfect fifth apart (on the above), compared with the harmonics of two notes a major second apart (on the below). The blue lines mark the harmonics that the upper and the lower notes have in common. The larger amount of common harmonics gives to the perfect fifth its consonant character; on the contrary, the tone is dissonant.]

Among the consonant intervals, the unison, the octave, the perfect fifth and the perfect fourth are also called *perfect consonances*, whereas the (major and minor) thirds and the (major and minor) sixths are *imperfect consonances*. Actually, the term "perfect consonance" don't be totally appropriate in the case of the perfect fourth interval: in fact, when the fourth is placed right above the lowest note of a chord (or when writing for two voices or instruments only), it may sounds dissonant. On the contrary, if the fourth separates two of the upper notes of a chord, it is consonant. This happens because the fourth's upper note doesn't belong to the initial harmonics of the lower note.

The diatonic scale comprehends a great amount of consonant intervals. In fact, there are six fifth intervals (C-G, D-A, E-B, F-C, G-D, A-E), three major thirds (C-E, F-A, G-B), four minor thirds (D-F, E-G, A-C, B-D), and respectively the same number of fourths (the fifth intervals inverted), minor sixths (the major thirds intervals inverted) and major sixths (the minor thirds intervals inverted). In addition, the whole length of the scale is the octave.

All the consonant and dissonant intervals can be found by looking at the distances between the initial harmonics, as it was anticipated in chapter 1. To be specific, the octave is the distance between the 1st and the 2nd harmonics, the perfect fifth is the distance between the 2nd and the 3rd, the perfect fourth between the 3rd and the 4th, the major third between the 4th and the 5th, the minor third between the 5th and the 6th. Moreover, the distance between the 3rd and the 5th is a major sixth, and the distance between the 5th and the 8th is a minor sixth.

Consequently, it is common to identify the intervals, instead of taking a couple of notes, by specifying the corresponding distance between the harmonics. To be specific, being multiples of the fundamental frequency, the harmonics have the property that the 1st harmonic is half the frequency of the 2nd, the 2nd is 2/3 the frequency of the 3rd, the 3rd is 3/4 the frequency of the 4th, and so on. This relationship can be used for calling the intervals by their **frequency ratio**: the interval between the 1st and the 2nd harmonics (the octave) corresponds to the 1:2 ratio, the interval between the 2nd and the 3rd harmonics (the fifth) corresponds to the 2:3 ratio, and so on. In general, the interval between the harmonic number *n* and the harmonic number *m* corresponds to the frequency ratio *n:m*.

5-Building the diatonic scale

The music of almost every culture has evolved around particular scales. The shape of the scales has been influenced by uncountable factors, such as the human perception of the involved intervals and the technological advancement of the adopted instruments. Thus, since a scale is the result of some human-dependent aesthetic and historical development, a purely acoustic analysis cannot completely explain its origins.

However, the acoustic phenomena being involved in the particular form of a scale contribute in justifying some aspects of the music that from the scale is produced. Moreover, such phenomena are related to the brain's capability of computing the relationships between the scale's notes, and thus they can clarify part of the processes that led to the preference of the scale over its variants that have appeared in the history and then have became obsolete.

The procedure which is usually adopted in the "scientific" derivation of a scale consists of considering some of its degrees and then, by looking at their first harmonics, to justify the presence of the other degrees. In addition to the harmonics, also the harmonics of the harmonics, called **secondary harmonics**, are taken into consideration. For being distinguished from the secondary harmonics, the firsts are also called *primary harmonics* or *generators*.

Not all the initial harmonics of the notes are available: some of them, in particular the 11^{th} and 13^{th}, are considerably out of tune, being their equal-temperament approximation quite large (respectively -49 ¢ and +31 ¢).

A frequently adopted derivation of the diatonic scale consists of considering the harmonics of the tonic, the dominant and the subdominant degrees. That is, in the case of the C diatonic scale, C, G and F. As explained e.g. in text *Theory of Harmony* by A. Schonberg, this choice is driven by the fact that G, the 3^{rd} harmonic of C, brings the strongest contribution to C because it is the first harmonic of C that is different from the octaves. Therefore, its harmonics are the most significant secondary harmonics in the sound of C. Furthermore, the harmonics of C can themselves be viewed as secondary harmonics: since C is the 3^{rd} harmonic of F, its harmonics are the most significant secondary harmonics of F.

Thus, consider the first 5 harmonics C, G and F:

C, C, G, **C**, **E**, …

G, G, **D**, **G**, **B**, …

F, F, C, **F**, **A**, …

All the diatonic notes are present. The bolded notes lie in the same octave.

For displacing them in the right order, one method is to directly look at their frequencies. However, the analysis can provide deeper results whenever a greater number of harmonics is taken into

consideration. Let's display the harmonics of C, F and G so that the 9th harmonic of F is aligned with both the 6th harmonic of C and the 4th harmonic of G (they all are the note G). Leaving empty some positions, one gets:

F: F F C F _ A **C** _ **Eb F G A** …

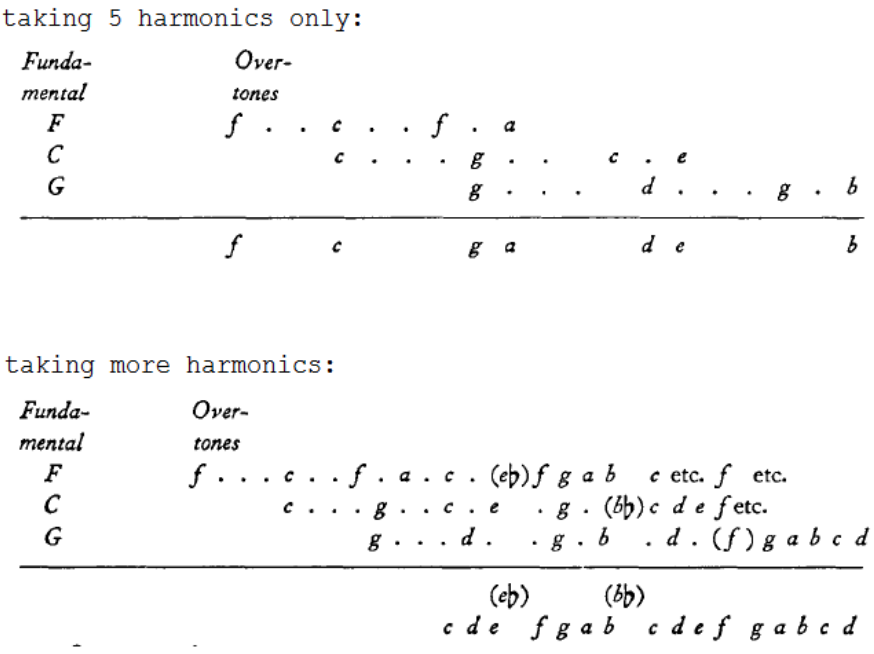
C: C _ C _ G _ **C** _ **E** _ **G** _ **Bb** C …

G: G _ _ _ G _ _ **D** _ _ **G** _ **B** _ _ D…

Then, look at the bolded letters. In all the three sequences:

- The 7th position is occupied by C (or it has been left empty)
- The 8th position is occupied by D (or it has been left empty)
- The 9th position is occupied by either E or Eb (or it has been left empty)
- The 10th position is occupied by F (or it has been left empty)
- The 11th position is occupied by G in all the sequences
- The 12th position is occupied by A (or it has been left empty)
- The 13th position is occupied by either B or Bb (or it has been left empty)

Observe the presence of both B and Bb: if proceeding with the following octave, where the harmonics of G have greater volumes, B is favored over Bb. Also, E is to be preferred in place of Eb because it's the 5th harmonic of C (that is, a primary harmonic of the tonic).



[**Figure 5.1**: Harmonics of F, C and G. Taken from Arnold Schoenberg's *Theory of Harmony*.]

Some authors justify the diatonic scale by considering, in place of the subdominant, the supertonic degree (in the C diatonic scale, D). With such selection, the scale is derived as follows:

- Tonic
- Supertonic
- Mediant: 5th harmonic of the tonic
- Subdominant: 7th harmonic of the dominant (secondary harmonic)
- Dominant
- Submediant: 3rd harmonic of the supertonic (secondary harmonic)
- Leading-tone: 5th harmonic of the dominant (secondary harmonic)

The two derivations don't be in opposition, and the aspects in which they differ provide further elements to be analyzed when considering the importance that the diatonic scale has acquired during the centuries.

6-Tonality and triads

The ability of the human brain to recognize the intervals between the notes is an essential aspect of the perception of the music. As noted in chapter 1, the brain doesn't have the capacity to identify the exact frequency of the notes (except for the people that have the absolute pitch): it only detects the intervals through the comparison of the harmonics of notes. If the brain perceives that the 1^{st}, 2^{nd}, 3^{rd}, … harmonics of a note coincide with the 2^{nd}, 4^{th}, 6^{th}, … harmonics of a second note, it recognizes the octave interval. If the brain perceives that the 2^{nd}, 4^{th}, 6^{th}, … harmonics of a note coincide with the 3^{rd}, 6^{th}, 9^{th}, … harmonics of a second note, it recognizes the perfect fifth interval.

The octave and the fifth require the smallest amount of computation for being identified, and therefore are perceived as the most consonant intervals. Vice-versa, when the brain finds a little quantity of coincident harmonics its elaboration becomes more difficult, and the effect of dissonance is produced. E.g., in the case of the tone interval, only the 8^{th}, 16^{th}, 24^{th}… harmonics of the upper note coincide with the 9^{th}, 18^{th}, 27^{th} … harmonics of the lower note.

Moreover, if the fundamental frequency is artificially subtracted from the harmonics of a note, it has been shown that the listener will continue to detect the same note, that is, he will continue to hear the non-existing 1^{st} harmonic. The brain processes the other harmonics, and spontaneously calculates the fundamental frequency. This phenomenon is called **virtual pitch**, and it is used in engineering e.g. when the speakers are too physically small to produce the lowest notes: instead they play only their harmonics, and rely on the brain's computation to reconstruct the note.

This suggests that, when a melody or a chord adopts notes that coincide with the harmonics of a fundamental frequency, or with its secondary harmonics (see the previous chapter), such fundamental frequency is to some extent required by the listener. Under the light of the virtual pitch, the acoustic affinity of the diatonic scale's tonic with the other degrees contributes to recognize the tonic as fundamental frequency, and to perceive the other notes in their relation with it. Consequently, during the time in which a melody or a chord doesn't comprehend the tonic, a more or less strong sense of instability and tension is produced. Then, when the tonic is finally sounded, it is felt as a resting point and provides a pleasant sense of conclusion.

In other words, the tonic pulls towards itself all the other notes: this is the concept of **tonality**. The tonality is the relationship between all the notes of a piece of music and one particular note, the tonic of the scale which is used. This relationship is established by the harmonics (and secondary harmonics) that the tonic has in common with the other degrees of its scale.

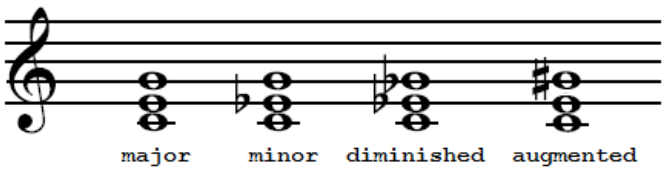
A tonality is called **major tonality** (also **major key** or **major mode**) if the major scale is adopted, and it's called **minor tonality** (**minor key**, or **minor mode**) if the natural minor scale (and its two variants, as discussed in the next chapter) is adopted. Since the notes of the major and the natural minor scales are the same, the establishment of either a major or a minor tonality requires that the primary role of the corresponding tonic degree (and its harmonics) is emphasized by the

composer. The tonic of the major or the natural minor scale is also called *tonic of the key* or *root of the key*. The **tonal music** consists of the music where a major or minor tonality is recognizable.

A displacement of three notes whose relative distance is a major or minor third interval is called **triad**. From this definition, it follows that four types of triad are available:

- The **major triad**, where a major third lies between the lowest note and the middle note, and a minor third is between the middle and the upper note. E.g., C-E-G is a major triad.
- The **minor triad**, where a minor third lies between the lowest note and the middle note, and a major third is between the middle and the upper note. E.g., C-Eb-G is a minor triad.
- The **diminished triad** is made up by adding two minor thirds. E.g., C-Eb-Gb is a diminished triad.
- The **augmented triad** is made up by adding two major thirds. E.g., C-E-G# is an augmented triad.

The lowest note (the bass) is called the **root** of the triad (the term "root" is also used for the bass of the other chords). The middle note is called "third" of triad and the upper note is the "fifth", because they respectively form a third interval and a fifth interval with the root. The four types of triad are shown in figure 6.1.



[**Figure 6.1**: The four types of triad.]

The root is the most important element of a triad. When the root of a major triad is the tonic of the major scale, and when the root of a minor triad is the tonic of the natural minor scale, the triad is called **tonic triad**. E.g., the tonic triad of the C major tonality (where the tonic is C) is the major triad C-E-G, and the tonic triad of the A minor tonality (where the tonic is A) is the minor triad A-C-E.

For establishing the tonality in piece of music, the tonic triad is the most important chord that have to be used. It is the sound that the listener recognizes as the main point of arrival where the musical phrases should end: this triad provides a sense of conclusion every time the music returns to it. As a consequence, is very frequent to have the tonic triad at the beginning and at the end of a piece of music.

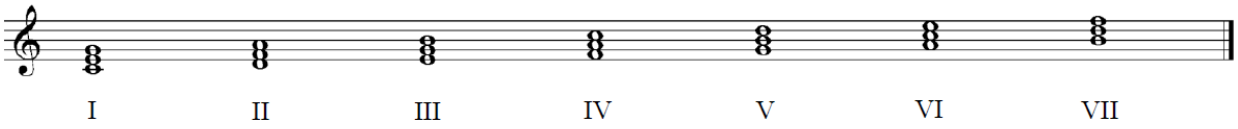
The fundamental role that the tonic triad plays in tonal music is related to the acoustic aspects of the major and minor triads. To be specific, the major triad is the best ambassador of the harmonics of its root: taking e.g. the C-E-G major triad, the third (E) and the fifth (G) coincide with the first two

harmonics of the root (C) that are different from its octaves. Thus, the major triad omits the more distant harmonics of its lowest note and reinforces the more immediate. Also the minor triad has analogous properties, and will be analyzed in the next chapter.

In the music of the common practice period, a large amount of chords are either major or minor triads, or can be interpreted as such. In particular, each degree of the diatonic scale can become the root of either a major, a minor or a diminished triad whose third and fifth correspond to two of the other degrees. To be explicit, let's consider the case of C major tonality. All the triads that can be produced by using the 7 notes of the C diatonic scale are the following:

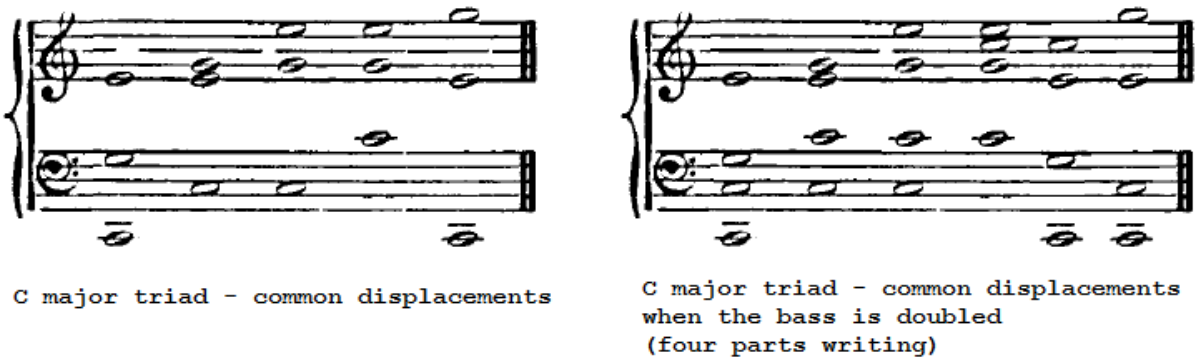
C-E-G ; **D**-F-A ; **E**-G-B ; **F**-A-C ; **G**-B-D ; **A**-C-E ; **B**-D-F

These chords are known as **diatonic triad**s. The triads C-E-G, F-A-C and G-B-D are major triads; D-F-A, E-G-B and A-C-E are minor triads; and B-D-F is a diminished triad. These triads also belong to the A minor tonality, since the diatonic scale is the same.



[**Figure 6.2**: The diatonic triads.]

Triads can appear in different forms. In fact, by transporting the third, or both the two upper notes, to an octave higher, then the triad's acoustic features and qualities don't change. Also, by duplicating any of its notes (more often the bass or the perfect fifth) onto the octaves above, the nature of the chord remains the same: it is in fact the ordinary practice when writing for more than three elements. In all these cases, the root remains the lowest note.



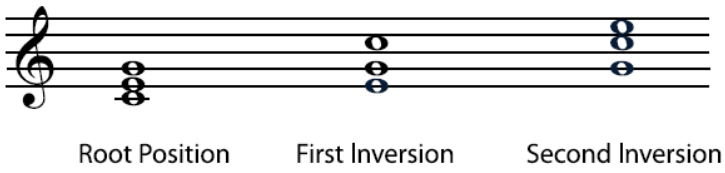
[**Figure 6.3**: Examples of C major triad's displacement in three and four parts writing.]

In the figure 6.3 some common solutions for placing the C major triad on the grand staff are shown, both in the case of three and four parts writing (in this case, the bass has been doubled). Not all the other possible displacements are equally good, as it will be discussed in chapter 10.

If the displacement of the triad's notes is altered so that the root doesn't be the bass, quite different chords arise: they are called **triad inversion**s, because involve the inversions of the intervals that the third and the fifth form with the root. To be specific, triads have 2 inversions (shown in figure 6.4):

- The chord which is obtained by putting the root on the octave above (or by putting the upper two notes on the octave below) is called **first inversion**. E.g., the first inversion of C-E-G is E-G-C, where the bass is E.
- The chord which is obtained by putting the lower two notes on the octave above (or by putting the fifth on the octave below) is called **second inversion**. E.g., the second inversion of C-E-G is G-C-E, where the bass is G.

Thus, the first inversion places the third in the bass, the second inversion does the same with the fifth. Observe that the inversion of an augmented triad produces another augmented triad.



[**Figure 6.4**: Inversions of the C major triad.]

The first inversion of a triad is also known as *sixth chord*, or *chord of the third and sixth*, because the distances between the bass and the other two notes are respectively a third and a sixth interval. Similarly, the second inversion is called *six-four chord*, or *chord of the fourth and sixth*, because a fourth and a sixth intervals are formed with the bass. Since the inversions don't change the consonant/dissonant character of the intervals, chords are very often replaced with their inversions. However, being less incisive, inversions are seldom used for concluding the musical phrases.

If a triad doesn't be inverted, is said to be in **root position**, because the root is the bass. When speaking of "C major triad", the letter "C" always refers to the root of the triad, even if the triad is in inversion. The distinction between the root position and its inversions is related to the primary role that the bass assumes in the chords. The influence of the bass on the notes above it, in fact, is a central element of tonal music: being the lowest note, the harmonics of the bass predominate over those of the other notes.

In general, thus, the pleasantness or solidity of the chords mainly depends on the affinity of the upper notes with the bass; that is, it depends on the amount of harmonics that the upper notes have in common with the lowest one. As discussed in chapter 8, this is the reason because the inversions of the major and minor triads sound weaker and more undecided than their root position.

It is frequent to indicate the triads by adopting the Roman numerals I, II, III, IV, V, VI, VII: these numerals specify the degree of the major or minor scale upon which a triad is built (i.e. they

indicate the root). For example, in C major, I is C-E-G; in A minor, I is A-C-E. The connection from e.g. V to I is written V-I. There are different styles for writing the numerals: writers in German use upper-case letters, writers in English often adopt upper-case letters for major and augmented triads, and lower-case letters for minor and diminished triads. Moreover, there is a quantity of additional signs for indicating e.g. if the triad is in root position or in inversion, if additional notes are placed, if the triad is subjected to chromatic alterations, and so on. This text will use only the upper-case Roman numerals.

Since the major triad is representative of the most important harmonics of its root, the relationship between the tonic and the other notes of the major and minor scales is to some extent reflected in the triads that find in these notes their root. In particular, consider that:

- The brain spontaneously tries to recognize a harmonics sequence among the notes that are listened.
- The tonic is the note that the listener perceives as the reference sound of the piece: hence, the brain will primarily try to recognize the harmonics sequence of the tonic.
- The 3rd harmonic of a note is the first harmonic to be different from the octaves of the note. If the fundamental frequency is the tonic, the 3rd harmonic is the dominant degree.
- Therefore, when listening to the dominant, the brain tends to consider it as the 3rd harmonic the tonic. More generally, it tends to consider the harmonics of G as secondary harmonics of C.

As a consequence, it is possible to identify in G a significant predisposition to move towards the tonic C. The same applies to the triad whose root is the dominant degree (i.e., V), which is called **dominant triad**. The major domain triad provides the most incisive connection to I, i.e. V-I. This progression reinforces the listener's perception of I as the tonic triad, and therefore it clearly establishes the tonality which is adopted.

Any note behaves to some extent like a dominant degree, since any note can always be thought as the fifth above a tonic, i.e. its harmonics can always be considered secondary harmonics. Thus, any note has the tendency to fall on the fifth below, and then also every major triad.

A change of the tonality in the middle of a piece is called a **modulation**. The modulations, that will be discussed in chapter 16, are of very frequent usage in the common practice music, especially the Romantic period. Since a change of the tonality corresponds to the adoption of a different diatonic scale, a different key signature is required.

The key signature comprehends the set of accidentals which are incorporated in the diatonic scale that is used in a piece of tonal music, except for the case of the C diatonic scale. The couples of major and minor tonalities that have in common the same diatonic scale, therefore, have identical key signatures, and the minor tonality is called **relative minor** of the major tonality. Vice-versa, the major tonality is called **relative major** of the minor tonality. By looking at the key signature, consequently, it's impossible to understand if either a major or minor tonality is used, and the examination of the score is needed.

On the other hand, the couples of major and minor keys that have the same tonic, such as C major and C minor, are reciprocally called **parallel major** and **parallel minor** (and their key signatures differ in three notes). So, C major is the parallel major of C minor, and vice-versa.

Since the C diatonic scale doesn't have accidentals, no accidentals are written in the key signature of C major and A minor tonalities. The key signature of the other tonalities is obtained by transposing the C diatonic scale higher or lower. Since there are 12 semitones in one octave, 12 transpositions are possible. Since each diatonic scale provides two tonalities, 24 tonalities are available.

E.g., the G major key (and its relative minor, E minor) has one accidental, F#, in the key signature, because in the diatonic scale that begins on G:

G, A, B, C, D, E, F#

a sharp on the seventh degree is present. To write Gb would be wrong, because the leading-tone forms a major seventh with the tonic G, and not a diminished octave.

Similarly, the transposition of the C diatonic scale to F produces:

F, G, A, Bb, C, D, E

where Bb allows to obtain the diatonic's fourth degree (writing A# would produce an augmented third above the tonic, and not the perfect fourth). Consequently, F major and D minor have Bb in the key signature.

The transposition to the remaining diatonic degrees, D, E, A, and B, follows in analogous manner (as the reader can check, they all involve only sharps). In the case of transposition to the non-diatonic degrees of the chromatic scale, the diatonic scale can be written either using sharps (and, eventually, double-sharps) or flats (or double-flats). E.g., the diatonic scale of Db major (Bb minor) is:

Db, Eb, F, Gb, Ab, Bb, C

and the diatonic scale of C# major (A# minor) is:

C#, D#, E#, F#, G#, A#, B#

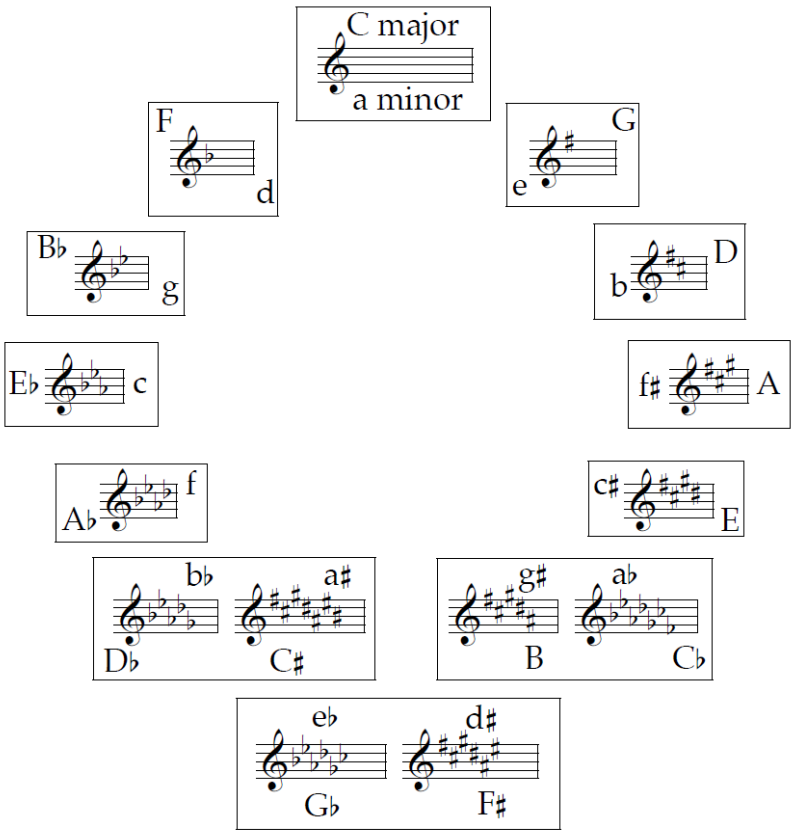
In the choice between C# and Db, D# and Eb, F# and Gb, G# and Ab, A# and Bb, the smaller number of accidentals which are produced in the transposition leads to prefer Db over C#, Eb over F#, Ab over G# and Bb over A#.

From the transposition of the diatonic scale, a general criterion can be recognized: taking the C diatonic scale as the starting point, the key signature acquires a sharp when transposing to the fifths above (G, D, E, B, F#) and acquires a flat when transposing the fifths below (F, Bb, Eb, Ab, Db, Gb). That is:

- The transposition to the fifth above C major (G major) produces F#, and all the other keys involving sharps also have F# in the key signature. The transposition to the fifth above G major (D major) produces, in addition to F#, the note C#, and all the remaining keys involving sharps also have both F# and C#. And so on…
- The transposition to the fifth below C major (F major) produces Bb, and all the other keys involving flats also have Bb in the key signature. The transposition to the fifth below F major (Bb major) produces, in addition to Bb, the note Eb, and all the remaining keys involving flats also have both Bb and Eb. And so on…

Therefore, while varying in number for differentiating the key signature, the accidentals follow a fixed sequence in their coming out. In the case of the sharps, the sequence is: F#, C#, G#, D#, A#, E#, B#. If a tonality comprehends e.g. A#, then all the preceding sharps in the list (F#, C#, G#, D#) are also present in its key signature. The same applies to the flats, but the order of appearance is reversed, that is, they follow the opposite sequence: Bb, Eb, Ab, Db, Gb, Cb, Fb. If a tonality comprehends e.g. Gb, then all the preceding flats in the list (Bb, Eb, Ab, Db) are also present in its key signature.

The keys that don't involve double-sharps and double-flats (i.e., the most frequently used ones) can be displaced in a circular relationship, called **circle of the fifths**, which is shown in figure 6.5.



[**Figure 6.5**: Circle of the fifths. Minor keys are indicated with lower-case letters.]

7-Minor tonality

The essential difference between the major and the minor tonalities is the quality of the third above the tonic: as previously noted, the tonic-mediant interval changes its amplitude whenever either the major or the natural minor scale is considered. In the major scale it is a major third (e.g., in C major, C-E), in the minor scale it is a minor third (in A minor, A-C).

The minor tonality is one of the most interesting elements of Western music: in fact, whereas the mediant degree of the major scale coincides with the 5^{th} harmonic of the tonic (E is the 5^{th} harmonic of C), the mediant degree of the minor scale (in C minor, Eb) doesn't appear neither between the initial harmonics of the tonic, nor between its initial secondary harmonics. Mostly because of this, many authors have considered the minor triad (and the minor tonality that it represents) artificial, in agreement with the Helmholtz's point of view, which regards the minor triad as "inferior" to the major triad. However, in the musical practice, the minor tonalities are diffused and appreciated as well as the major tonalities: the minor triad doesn't be perceived less consonant than the major triad, it simply sounds different.

The analysis of the harmonics involved in the minor triad may help to clarify this point. Consider the C minor tonic triad, C-Eb-G. The 3^{rd} and the 5^{th} harmonics of Eb are respectively Bb and G: Bb coincides with the 7^{th} harmonic of C, and G coincides with both the 3^{rd} harmonic of C and the 1^{st} (2^{nd}, 4^{th}, 8^{th}, ...) harmonic(s) of G. Therefore, the introduction of Eb in the sound of C and G is sustained by the two most important harmonics of Eb after its octaves.

Moreover, looking at the C natural minor scale:

C, D, Eb, F, G, Ab, Bb, (C)

it is possible to observe that both Bb and Ab (which don't be included in the C major scale) give further emphasis to Eb: in fact, Bb is the 3^{rd} harmonic of Eb, and Eb is the 3^{rd} harmonic of Ab.

It is to be noted, however, that E, being the 5^{th} harmonic of C, inevitably provides a relevant contribution to every triad whose root is C. Consequently, this harmonic may be in conflict with the presence of Eb in the C minor triad, especially when the root is doubled. This acoustic impurity, perhaps, gives to the minor mode its characteristic flavor.

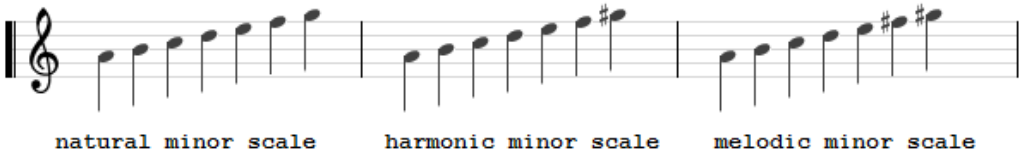
A typical aspect of the minor tonalities is the usage of chromatically rising the seventh degree, or both the seventh and the sixth degrees, of the natural minor scale, so that two additional scales arise. They are the **harmonic minor scale**:

A, B, C, D, E, F, G#

and the **melodic minor scale**:

A, B, C, D, E, F#, G#

which are shown in figure 7.1.



[**Figure 7.1**: The minor scales.]

Since F# and G# have fewer harmonics in common with the tonic triad A-C-E than F and G, these chromatic changes mainly have melodic purposes, as it will be discussed in chapter 11. To be specific, G# assumes an "harmonic function" only in particular situations (as shown in chapter 15).

The introduction of F# and G# leads to the birth of new triads, which enlarge the collection that was found on the diatonic scale degrees in the previous chapter. In the case of the harmonic minor scale, all the triads that is possible to obtain are:

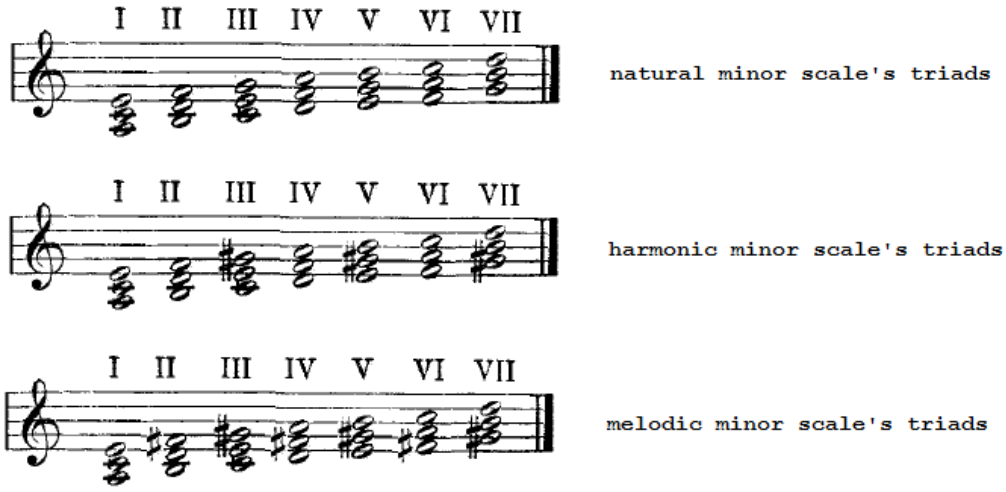
A-C-E ; **B**-D-F ; **C**-E-G# ; **D**-F-A ; **E**-G#-B ; **F**-A-C ; **G#**-B-D

The triads on the first and the fourth degrees are minor, the triads on the fifth and the sixth degrees are major, the triads on the second and seventh degrees are diminished, and the triad on the third degree is augmented.

In the case of the melodic minor scale, the triads are the following:

A-C-E ; **B**-D-F# ; **C**-E-G# ; **D**-F#-A ; **E**-G#-B ; **F#**-A-C ; **G#**-B-D

where the triads on the first and second degrees are minor, the triads on the fourth and fifth degrees are major, the triads on the sixth and seventh degrees are diminished, and the triad on the third degree is augmented.



[**Figure 7.2**: Triads on the minor scales.]

These scales produce the augmented triad, which is absent among those obtained from the degrees of the diatonic scale. The augmented triad divides the octave in three equal parts; it will be discussed in chapter 17.

It is to be remarked that the origin of a chord from the minor key, such as the augmented triad, never obstacles its use in major keys. The derivation from a scale is only a abstract procedure that tries to explains the closeness of the chord to the aesthetic (acoustic and/or historical) elements that are involved in the shape of the scale, and never limits the artistic possibilities that the chord can supply.

8-Inversions of major and minor triads

As discussed in chapter 6, triads have two inversions: in the first inversion the third is placed in the bass, while in the second inversion the fifth is the lowest note. The harmonics of the bass provide a greater contribution to the chords than that of the other notes; therefore, in the case of the major and minor triads, a change in the bass alters the triad's stability and solidness: the acoustic aspects that make the root position particularly solid lose much of their influence.

To be specific, whereas in the major and minor triads' root position the third and the fifth have numerous harmonics in common with the bass, if the bass changes then also the harmonics that have to be considered change. When the triad is inverted, the bass forms different intervals with the other two notes (in the first inversion, a third and a sixth; in the second inversion, a fourth and a sixth). As a result, the number of the harmonics that it has in common with the upper notes decreases.

However, since the inversion of a consonant interval produces another consonant interval, dissonances cannot arise from the inversion of a major or minor triad (except for the fourth's special behavior: as it was noted in chapter 4, the fourth with the bass produces the effect of a dissonance). Therefore, the inversions can be considered "weaker" representatives of the corresponding root positions.

The inversions of the major and minor triads have a number of useful characteristics. In fact, the lesser affinity of the new bass with the upper notes subtracts to the bass the emphasis the its lowest position would spontaneously provide. This weakening of the bass line lighten the music of the other parts. Inversions also produce variety in the chords displacement because consent to avoid the repetition of the same lowest note. Moreover, they offer many additional possibilities in the design of the voice-leading, and thus allow the composer to obtain smoother chord progressions, as it will be discussed in chapter 10.

In the evaluation of the acoustic properties of the inverted triad, the change in the bass implies that different harmonics have to be taken. To be specific, considering the C major triad C-E-G, the root position involves the following harmonics:

C: C_ _ _ C_ G_ C_ E_ G …

E: _ **E**_ _ _ **E**_ _ B_ **E**_ G# …

G: _ _ **G**_ _ _ _ **G**_ _ D_ _ **G** …

where the bolded letters mark the harmonics that the upper notes have in common with the bass. Then, consider the first inversion:

E: E_ _ _ E_ B_ E_ G#_ B …

G: _ G_ _ _ G_ _ D_ G_ **B** …

C: _ _ C_ _ _ _ C_ _ G_ _ C_ **E** …

and the second inversion:

G: G_ _ _ G_ D_ G_ B_ D …

C: _ C _ _ _ C_ _ **G**_ C_ E …

E: _ _ E_ _ _ _ E_ _ **B**_ _ E …

As it can be noted from the scarcity of bolded letters, in the two inversions the bass gets lesser sustainment from the upper notes. In particular, in the case of the second inversion, the harmonics which are shared with the bass (G and B) are closer to the beginning of their sequences than those of the first inversion, and therefore they usually have greater volumes. This gives the second inversion a slight greater affinity with the root position. However, such advantage is obscured by the presence of a fourth interval between the bass and the root.

9-Consonance and dissonance

The acoustical relationship between the two notes the form the intervals is established by the harmonics that these notes have in common. As noted in the previous chapters, the harmonics provide a consonant or a dissonant character to the intervals. In particular, the initial harmonics of a note are the most significant: the first 8 harmonics allow to find (by looking at their relative distances) all the consonant intervals which are used in Western music.

It is possible to subdivide the consonant intervals into 2 groups:

- The intervals between the fundamental frequency (or its octaves) and the first 5 harmonics. That is, the unison (the interval between the fundamental frequency and itself), the octave (between the fundamental and the 2nd harmonic), the perfect fifth (between the fundamental and the 3rd harmonic) and the major third (between the fundamental and the 5th harmonic).
- The intervals that don't comprehend the fundamental, that is, the perfect fourth (the interval between the 3rd and the 4th harmonics), the minor third (between the 5th and the 6th), the major sixth (between the 3rd and the 5th), and the minor sixth (between the 5th and the 8th).

Thus, whereas in the first group the intervals' upper note is itself one of the 5 initial harmonics of the lower note, in the second group, on the contrary, the same relationship doesn't take place. E.g., if the lower note is C, the fourth interval is C-F, the minor third is C-Eb, the major sixth is C-A, and the minor sixth is C-Ab: all their upper notes, F, Eb, A and Ab, don't belong to the initial harmonics of C.

The second group (which comprehends the inversions of the intervals in the first group, plus the minor third and the major sixth) is representative of a consonant behavior which is different, and perhaps to some extent weaker, than that of the first group. In fact, the consonance of the intervals in the second group is provided by the following relationship: either the 3rd or the 5th harmonic of the upper note coincides with either the 1st, the 3rd or the 5th harmonic of the lower note. To be specific:

- In the fourth interval C-F, the 3rd harmonic of F is C, which is the 1st harmonic of C.
- In the minor third interval C-Eb, the 5th harmonic of Eb is G, which is the 3rd harmonic of C.
- In the major sixth interval C-A, the 3rd harmonic of A is E, which is the 5th harmonic of C.
- In the minor sixth interval C-Ab, the 5th harmonic of Ab is C, which is the 1st harmonic of C.

This analysis might allow to consider the intervals in the first group "tonically consonant", because their upper note coincides with one of the initial 5 harmonics of their lower note, whereas those in

the second group can be viewed as "phonically consonant", because the harmonics that the upper note has in common with the lower note are either the 3rd or the 5th, but not the 1st.

In the case of dissonant intervals, on the other hand, neither the upper note nor its first 5 harmonics coincide with one of the first 5 harmonics of the lower note (i.e., they don't belong neither to the first group nor to the second group). In fact:

- In the minor second interval (C-Db), the lower and the upper notes don't have harmonics in common (among the initial ones).
- In the major second interval (C-D), D is the 9th harmonic of C, the 7th harmonic of D is C (1rd harmonic of C), and the 9th harmonic of D is E (5th harmonic of C).
- In the tritone interval (C-F#), the 7th harmonic of F# is E (5th harmonic of C).
- In the minor seventh interval (C-Bb), Bb is the 7th harmonic of C, the 5th harmonic of Bb is D (9th harmonic of C), and the 9th harmonic of Bb is C (1st harmonic of C).
- In the major seventh interval (C-B), the lower and the upper notes don't have harmonics in common (among the initial ones).

When referring to the "initial harmonics", it would be useless to consider those beyond the 9th: the 10th and the 12th are respectively octaves of the 5th and the 3rd; the 11th and the 13th are out of tune (large equal-tempered approximations are needed), and the influence of the next ones, in any case, is greatly lessened by the decreasing of the harmonics' volumes.

Observe that this analysis doesn't provide explanation to the dissonance of the augmented and diminished intervals whose amplitude corresponds to that of consonant intervals. In fact, the augmented or diminished nature of an interval is defined by the interval's relationship with the diatonic scale. Whereas the "absolute" consonance or dissonance of an interval depends on its amplitude, in tonal music all the notes are bound to the tonic and its harmonics, and this rapport is reflected into the form of the diatonic scale. When the intervals involve notes that don't belong to the diatonic scale, such notes are (to some extent) in conflict with the tonality that have been established.

E.g., the intervals A-F and C-Ab have the same amplitude (a minor sixth): if they are used in C major tonality (or A minor), the first is consonant and the second is dissonant. On the contrary, when they are used in Eb major tonality (or C minor), the first is dissonant and the second is consonant. In fact, in C major, A-F is always minor sixth, while C-Ab could also be written as C-G#, which is an augmented fifth. On the contrary, in Eb major, C-Ab is always a minor sixth, because Ab is in the key signature (i.e. it arises from the transposition of the C diatonic scale), while A should be written as Bbb, and Bbb-F is an augmented fifth (though, in practice, in C major A is often written through a natural sign).

In Western music, the dissonant intervals are fundamental ingredients. The art of writing tonal music involves the ability to create climaxes and progressions, so that the listener is continuously waiting for something to come. When a sequence of chords generates an increasing effect of

tension and stress, in the listener spontaneously arises the need for some stratagem to release such energy. The elements providing tension are the dissonant intervals.

Chords including dissonant intervals are called **dissonant chord**s or **discord**s, and the word "dissonance" can refer to both dissonant intervals and dissonant chords. Since the harmonics of the bass prevail over those of the other notes, the effect of dissonance is emphasized when the dissonant interval involves the bass. The chords involving consonant intervals only, called **consonant chord**s (substantially, major and minor triads), are the targets where the dissonance's tension is discharged and leaves the place to relaxation and stability.

It is to be remarked that the dissonances don't correspond to bad sounds: a dissonant chord is a chord creating tension, it provides dynamics. If music involved consonant chords only, it would be static, mawkish and sloppy.

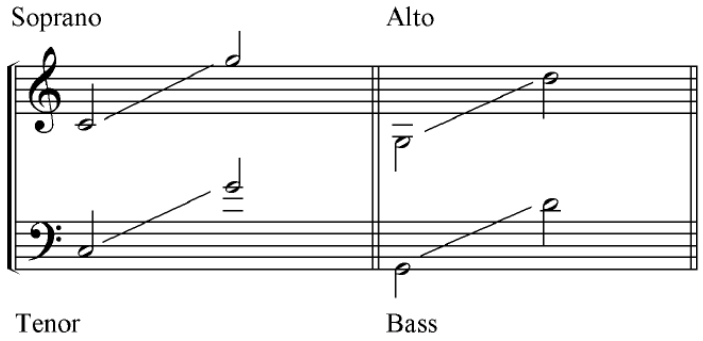
10-Voice-leading

In the previous chapters some aspects of the vertical dimension of the music have been discussed. However, the intervals and the chords are entities resulting from the contemporaneous movement of the parts. Therefore, the connections between the chords are also governed by melodic (horizontal) events. The laws of the voice-leading reflect the manifestation of such melodic influences in the relationships between chords.

The basic implementation of the principles of the voice-leading consists of displacing a bare succession of chords: each note of the chords is assigned to a part (instrument or voice). In this simple layout, the melodies of the parts are forced to change simultaneously, eventually taking the same note of the previous chord.

The traditional voice-leading is roughly based on the 4-part chorale practice of J.S. Bach. It applies to both vocal and instrumental music, though the historical evolution has taken place in the context of the choral liturgical music, where the choir elements usually sang at sight and without the help of instruments providing reference notes or chords. As a result, the following guidelines consent to obtain complex works through the superposition of simple melodic lines. The writing for three, five or more parts substantially follows from the same principles of the writing for four voices.

The four-part chorale-style format is notated on two staves, with each staff containing two parts: soprano and alto voices on the upper staff, tenor and bass voices on the bottom staff. Soprano and tenor are written with stems up, the others with stems down. Each part is restricted in its limited range (see figure 10.1) and the crossing between voices (one voice goes higher than one of its neighboring voices) should be avoided.



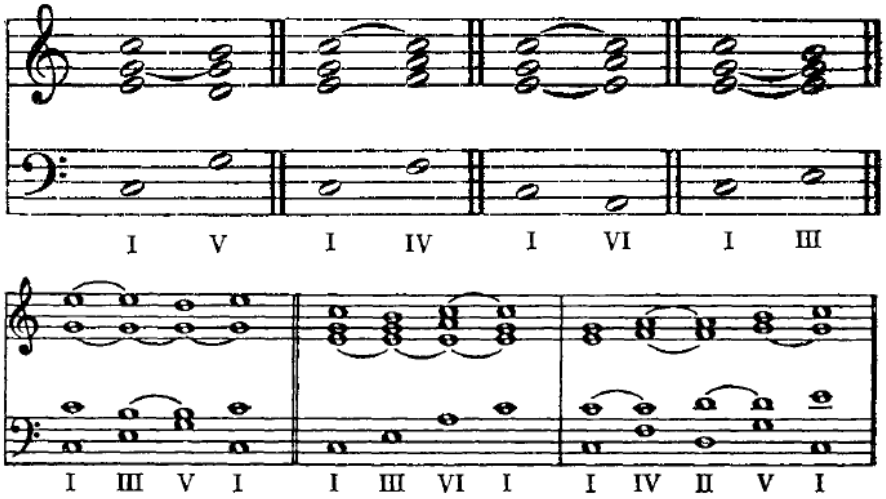
[**Figure 10.1**: Voice ranges in the four-part writing.]

The highest part's melody is perceived with greater clearness than the lower voices because of the higher pitch itself. Therefore, it is to be considered the song's leading melody. Behind the higher part, also the bass is easily identified by the listener, and thus assumes a secondary melodic function (in fact, it is sometimes indicated as the "second melody"). The highest and the lowest parts are called the *outer parts*, while the remaining ones (lying in the middle) are the *inner parts*.

As a general rule for to obtain fluent and homogeneous chords connections, the inner parts should be the most stationary as possible. They should move only if they really have to, making no more than the actions which are absolutely needed for connecting the chords.

When two consecutive chords have a note in common, this note should continue to stay in the same part. When a note remains sustained during the transition from one chord to the following, it is called a **common tone**.

Common tones can be viewed as a very powerful glue for connecting different chords. Observe that if a note is stationary while the other parts change, then also its harmonics continue to live. Thus, a greater number of notes are actually shared between consecutive chords that involve common tones. E.g., the tonic's triad shares two notes with both the triad on third degree and the triad on the sixth degree; and shares one note with the triads on the fourth and on the fifth degrees. Therefore, while performing e.g. the connections I-III, I-IV, I-V and I-VI, the common tones should be sustained (see figure 10.2). The use of chords' inversions considerably enlarges the set of the available possibilities for allowing the common tones to belong to the same part.



[**Figure 10.2**: Examples of common notes tying.]

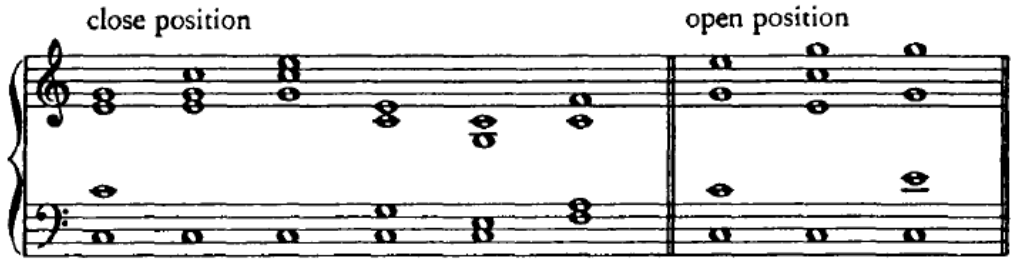
Whenever no common notes are available, and either a step or half-step motion is not practicable, each part should avoid leaps through other dissonant intervals. In particular, augmented ascending leaps are worse than the corresponding descending diminished leaps. While the inner parts should most frequently move by semitones or tones, and rarely jump more than a fifth, the bass can span larger intervals. Also the highest part is allowed to freer stunts because of the aesthetic purposes that its melody is equipped with. When a voice have to make big leaps, a motion in the opposite direction should follow. Moreover, it is advisable that sum of two consecutive leaps (in the same direction) doesn't be a dissonant interval.

When writing triads for more than three parts, it arises the necessity to assign a note to each part by putting the same note in two (or more than two) parts, in unison or at different octaves. In the

case of the major and minor triads, the preferred note to be duplicated is the root; as a second choice, the fifth. The doubling of the middle note (both in major and minor triads) produces a weaker result, though it also sounds good and it is very frequent. When dealing with dissonant chords, the duplication of the notes forming the dissonant intervals increases the dissonance because it produces an inversion or a replica of the dissonant interval.

Although it doesn't exist a rigid criterion for displacing the notes through the vertical dimension, for obtaining a balanced sound, in general, the smallest intervals between the notes should lie sufficiently high in the staff, so that the upper voices are more often spaced closer to each other than the lower ones. In particular, the upper three voices should be kept within an octave of one another, and the presence of the intervals smaller than the fifth in the low registers often sounds bad, even if they are consonant (especially if the upper parts are distant from the two lowest ones). This can be related to the natural displacement of the harmonics, where the higher elements are separated by smaller intervals than the first ones.

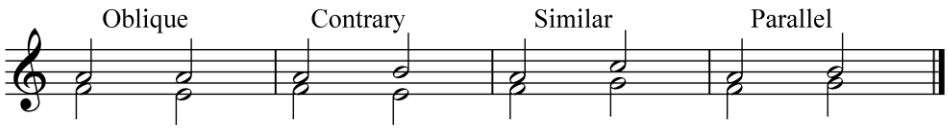
It is possible to identify two main styles that should be roughly followed: the *close position* and the *open position* (see the figure 10.3). When a triad is written in close position, no other triad's notes can be inserted between two adjacent parts. The close position is related to a brilliant and clear sound; if a spreader displacement is used, the sound becomes soft and mellow.



[**Figure 10.3**: Close position and open position.]

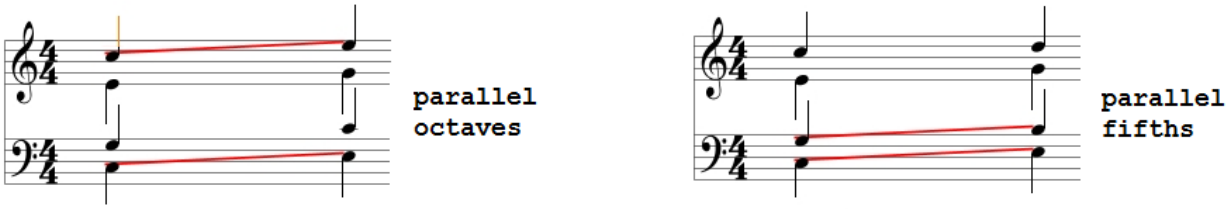
Since chords arise from the superposing of different and autonomous melodic layers, a very general principle of the harmony is to maintain the **relative independence of the parts**. That is, the voices should differentiate their relative motions. It is possible to distinguish four types of collective motion: **contrary motion**, **oblique motion**, **similar motion** and **parallel motion** (see figure 10.4). The autonomy of the parts is obtained by avoiding a frequent use of parallel motion (in particular, no more than two parts can go parallel). Contrary motion of the upper voices to the bass produces good results.

To be specific, the parallel motion in octave, fifth and also (to a lesser extent) the fourth is to be avoided unless a big amount of parts are present. In the case of the octaves and the fifths, the parallel movement is respectively called **parallel octaves** and **parallel fifths** (shown in figure 10.5): in the four-part writing it always sounds harsh. On the contrary, parallel thirds and sixth are regularly used in the music of the common practice.



[**Figure 10.4**: Examples of contrary, oblique, similar, and parallel motion.]

In the case of big ensembles, however, many elements will have to perform the same part, which usually must be pitched at different octaves for matching with the instruments' ranges. Therefore, in this context, parallel octaves are largely used.



[**Figure 10.5**: Parallel octaves, parallel fifths, consecutive octaves, consecutive fifths.]

When the music goes to a chord including a dissonant interval, one of the interval's notes should appear in the previous chord too - where it forms consonant intervals with the other notes. Then it remains held in the same part, so that when passing to the second chord it creates the dissonant interval without moving. That is, the note should be a common tone. This procedure is called **preparation of the dissonance**, and the note which remains sustained is said to be *prepared*.

This practice was introduced also because it helps the singers: the dissonant intervals are usually difficult to sing at sight, so that putting the problematic note in the chord before makes possible to sing it when it's easy to intone. This procedure for introducing dissonances can be iterated through a progression of dissonant chords (that is, the chord where the dissonant interval is prepared can be dissonant).

Since dissonances create a sense of instability, they need a target where to release the tension that they have accumulated. The definitive conclusion of a dissonant progression is called **resolution of the dissonance**, and must be a consonant chord. The dissonance is then said to be *resolved*. Usually, when going to the chord of resolution, the prepared note moves a semitone or a tone downward, or also (less frequently) a semitone or a tone upward, while the other note proceeds in the opposite direction or remains sustained. The resolution of the dissonance can be delayed: other chords intervene between the dissonant interval and the final consonant chord.

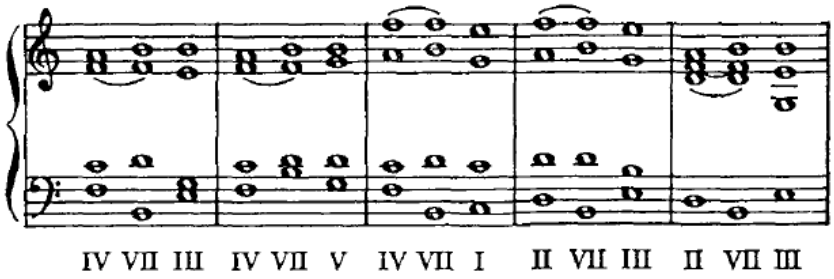
Both the preparation and resolution of dissonances should be performed by the same part, which begins and concludes into consonant intervals, and produces a dissonance only when it doesn't move. Moreover, doubling a dissonant note has to be avoided (also) because both the note and its replica should resolve the same way, creating parallel octaves.

As discussed in chapter 6, within any note lies the predisposition to fall a fifth below it (or to go up a fourth, which is the same), because any note can be viewed as the 3rd harmonic of the note lying a fifth below it. Accordingly, the ideal chord where the dissonance finds its resolution is the major or minor triad whose root lies a fifth below the root of the dissonant chord.

Among the four types of triad, two dissonant examples are present: the diminished triad and the augmented triad. They don't include the fifth interval, which is replaced by dissonant elements: the diminished triad has the tritone, and the augmented triad has the augmented fifth (diminished fourth when inverted). When resolving these dissonances, as it will be discussed in chapter 15, the notes forming diminished intervals have the tendency to approach one another, and notes forming augmented intervals have the tendency to diverge.

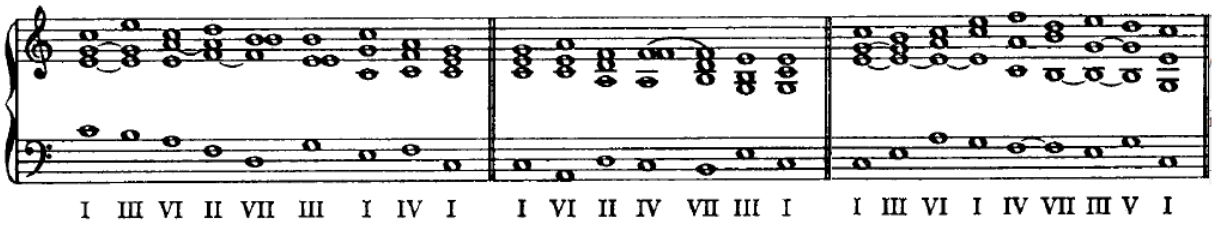
In the early times of tonal music, also the fourth above the bass (appearing e.g. in the second inversion of the major and minor triads) was treated like it was a true dissonance: either the bass or the fourth was prepared, and a common resolution was letting the fourth to become the root of the next chord.

In the figure 10.6 examples of treatment of the diminished triad's dissonance are shown.



[**Figure 10.6**: Examples of treating the tritone dissonant interval when appearing in the diminished triad VII of the major scale. The note F of VII is prepared and resolved.]

Figure 10.7 shows some examples of phrases where the guidelines of the traditional voice-leading in major apply.



[**Figure 10.7**: Phrases in major from Arnold Schoenberg's *Theory of Harmony*.]

During the common practice period, the above method for leading the parts through the dissonances has been progressively weakened; since the ending of the sixteenth century, various exceptions have arisen, and the treatment of dissonance has become more freer. If a dissonant

interval is introduced without preparation, then the two notes should enter by contrary motion, making the smallest possible movement (semitone or tone).

11-Traditional voice-leading in minor tonality

The guidelines that have been introduced in the previous chapter fully apply to both the major and the minor tonalities. In the case of minor tonalities, however, the harmonic and the melodic minor scales introduce new difficulties, since they comprehend chromatic altered notes.

As shown in chapter 7, in A minor key the harmonic minor scale is A, B, C, D, E, F, G#, and the melodic minor scale is A, B, C, D, E, F#, G#. These scales find their origin during the modal era, when the chromatic raising of either the seventh or both the seventh and the sixth degrees of the minor-like modes was typical, especially in the Dorian and the Aeolian modes. Such modifications happened spontaneously, being driven by the need of reaching the beginning note of the scale through an ascending motion of a half-step. Thus, the seventh degree was chromatically raised so that the leading-tone appears within the modes that don't have it.

As it will be discussed in chapters 15 and 16, the leading-tone has the tendency to lead the melody to the above tonic. The opposite motion (from tonic to leading-tone), on the contrary, doesn't be spontaneous to the same extent. Therefore, whereas the chromatic change in the seventh degree occurs especially when the melody ascends to the tonic, when descending from the tonic the seventh degree often remains unaltered.

The chromatic raising of the sixth degree is a consequence of the change in the seventh degree: if only the seventh was raised, in the ascending motion by steps to the tonic an augmented second leap (F-G#) would appear between the unaltered sixth and the altered seventh. The augmented second is dissonant and it is difficult to intone; hence, it should be avoided. Moreover, the F-G# interval produces a particular "oriental" flavor: it is included in the harmonic minor scale, and in fact such scale is the basis of many non-Western music.

Thus, the melodic minor scale is used for ascending to the tonic, and it is also called *ascending minor scale*. Vice-versa, the natural minor scale is also called *descending minor scale*, since it's more often used for descending from the tonic. The presence of different roads for approaching and leaving the tonic produces the following traditional rules (given in the case of A minor), which apply to all the parts' melodies:

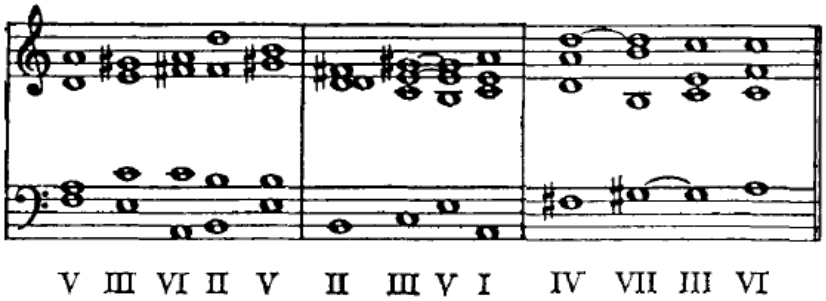
- If the melody takes G#, then it should ascend to A.
- When the melody takes G, it should descend to F (or it can leap).
- If the melody takes F#, then G# (and subsequently A) should follow.
- When the melody takes F, then G or E should follow (or it can leap).

Within these guidelines lies a general principle of the traditional voice-leading: the same part shouldn't move from a note to the adjacent non-diatonic note, that is, the melody shouldn't take half-step motions from a diatonic note to a chromatically altered note. In particular, a note cannot be followed by its chromatic alteration: in A minor, A can be occasionally joined with G#, but G never goes to G#, and F never goes to F#. This rule derives from the difficulty to intone the

semitone intervals, and forbids the possibility to use segments of the ascending (or descending) chromatic scale. However, the connections by half-steps actually produce very melodic results: the **chromatic voice-leading**, which belongs to the following evolution of Western music, is based on this possibility. It will be discussed in chapter 18.

The tendency of the leading-tone to bring the melody to the above half-step is a central aspect of harmony. In the non-chromatic voice-leading, every chromatically raised note is treated like it was the seventh or the sixth degree of a melodic minor scale: thus, a raised note should go either a semitone upward to a diatonic note (like the seventh degree) or a tone upward to another chromatically raised note (like the sixth degree). Vice-versa, every chromatically lowered note is considered the sixth or the seventh degree of a natural minor scale, and therefore it should go either a semitone downward to a diatonic note or a tone downward to another chromatically lowered note.

In figure 11.1 examples of voice-leading in A minor are shown.

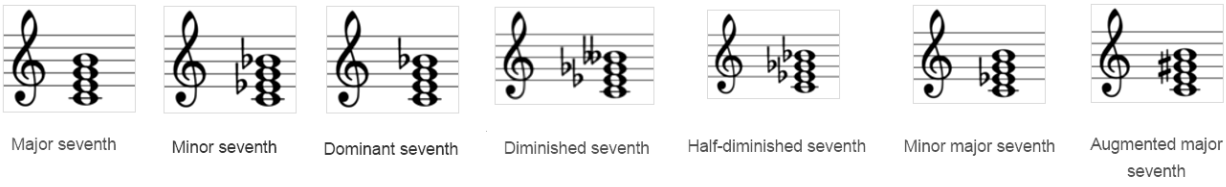


[**Figure 11.1**: Examples of the use of chromatic alterations of the sixth and seventh degrees in minor.]

12-Seventh chords

Among the chords of four notes, the seventh chords constitute a very important subset. They are obtained by placing another third above the triads, so that a seventh interval arises between the triads' root and the additional note. Therefore, they contain the root, the third, the fifth and the (major, minor or diminished) seventh. To be specific:

- If a major third is placed upon a major triad, a **major seventh chord** is produced. E.g., C-E-G-B. It comprehends one dissonant interval, the major seventh C-B.
- If a minor third is placed upon a major triad, a **dominant seventh chord** is produced. E.g., C-E-G-Bb. It comprehends two dissonant intervals: the minor seventh C-Bb and the diminished fifth E-Bb.
- If a major third is placed upon a minor triad, a **minor-major seventh chord** is produced. E.g., C-Eb-G-B. It comprehends two dissonant intervals: the major seventh C-B and the augmented fifth Eb-B.
- If a minor third is placed upon a minor triad, a **minor seventh chord** is produced. E.g., C-Eb-G-Bb. It comprehends one dissonant interval, the minor seventh C-Bb.
- If a major third is placed upon a diminished triad, a **half-diminished seventh chord** is produced. E.g., C-Eb-Gb-Bb. It comprehends two dissonant intervals: the minor seventh C-Bb and the diminished fifth C-Gb.
- If a minor third is placed upon a diminished triad, a **diminished seventh chord** is produced. E.g., C-Eb-Gb-Bbb. It comprehends three dissonant intervals: the diminished seventh C-Bbb and two diminished fifths, C-Gb and Eb-Bbb.
- If a minor third is placed upon an augmented triad, an **augmented-major seventh chord** is produced. E.g., C-E-G#-B. It comprehends two dissonant intervals: the major seventh C-B and the augmented fifth C-G#.



[**Figure 12.1**: The 7 seventh chords: major, minor, dominant, diminished, half-diminished, minor-major, augmented-major.]

The notes of the C diatonic scale produce the following seventh chords:

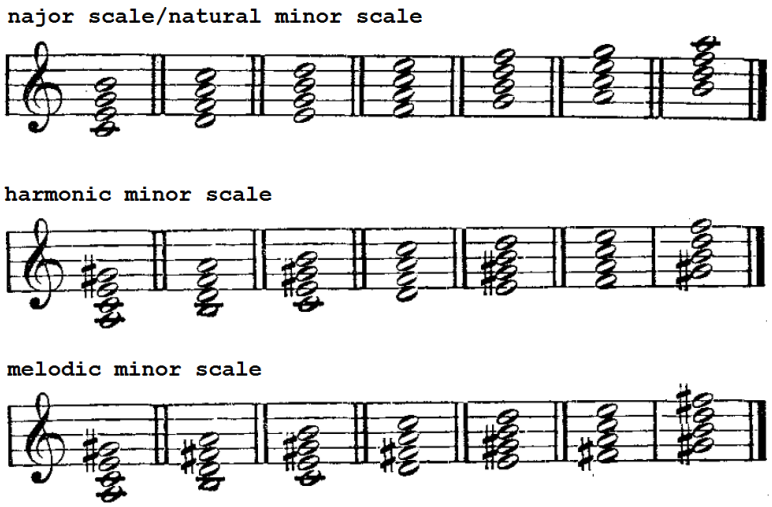
- **C**-E-G-B, a major seventh chord.
- **D**-F-A-C, a minor seventh chord.
- **E**-G-B-D, a minor seventh chord.
- **F**-A-C-E, a major seventh chord.
- **G**-B-D-F, a dominant seventh chord.

- **A**-C-E-G, a minor seventh chord.
- **B**-D-F-A, a half-diminished seventh chord.

Taking the A harmonic minor scale, the presence of G# generates a minor-major seventh chord (**A**-C-E-G#), an augmented-major seventh chord (**C**-E-G#-B), a dominant seventh chord (**E**-G#-B-D) and a diminished seventh chord (**G#**-B-D-F).

Taking the A melodic minor scale, the presence of F# and G# generates a minor seventh chord (**B**-D-F#-A), a dominant seventh chord (**D**-F#-A-C), and two half-diminished seventh chords (**F#**-A-C-E and **G#**-B-D-F#).

Hence, all the 7 seventh chords can be obtained from the major and the minor scales without introducing further accidentals, as shown in figure 12.2.



[**Figure 12.2**: Seventh chords on the major and the minor scales.]

The scale's degree upon which the seventh chords are built is called *root*, analogously to the case of the triads. When the root is the bass, the chords are in root-position. Since the seventh chords have four notes, there are three possible inversions: if the third is placed in the bass, the chord is called a *six-five chord*; when the fifth is the bass, it is called a *four-three chord*; when the seventh is the bass, a *two chord*. As in the case of the triads, these names refer to intervals which are formed with the new bass.

Among the 7 seventh chords, the dominant seventh chord is particularly important. In fact, its notes coincide with the first four new harmonics of the root, i.e. the 1st, 3rd, 5th and 7th harmonics (the 2nd, 4th and 6th harmonics are the octaves of the 1st and the 3rd). E.g., if the root is C2, then its harmonics are:

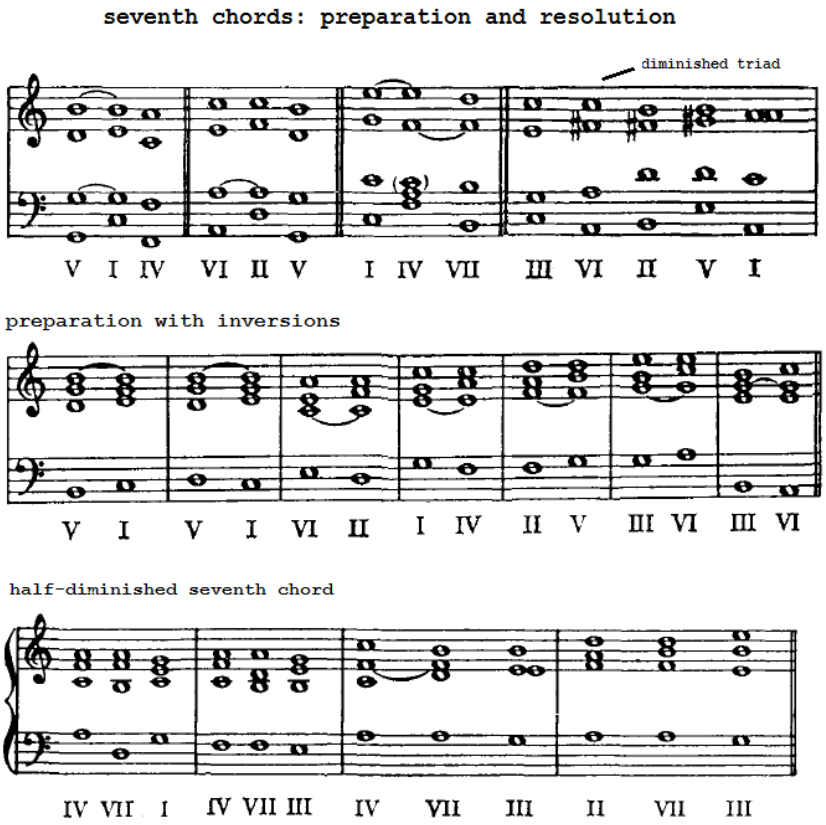
C2 C3 **G3** C4 **E4** G4 **Bb4** C5 …

where the bolded letters mark the dominant seventh chord C-E-G-Bb.

The root of the dominant seventh chord that can be obtained from the diatonic notes is the dominant degree. E.g., in C major, the dominant seventh chord is G-B-D-F. If the root is omitted, then the diminished triad VII (in C major, B-D-F) is obtained. Consequently, the diminished triad can be thought as a triad with two roots: the additional root is the root of the dominant seventh chord. Accordingly, one the common resolutions of VII is the chord whose root lies a fifth below (a fourth above) the root of the dominant seventh chord, i.e. the tonic triad.

The seventh chords usually resolve their dissonant seventh interval by letting the seventh to move one step downward, often becoming the third or the fifth of the next chord. It is also frequent to keep the seventh sustained while the bass ascends. In general, the resolution of a seventh chord into a consonant chord with the same root doesn't sound good, while the previous chord can be the same chord without the seventh (in such case the seventh is evidently unprepared). Also the connection of two seventh chords that have the same root is generally avoided.

The earliest laws of harmony would also forbid the resolution of the seventh interval into the octave: i.e., a seventh whose lowest note descends a step while the upper note is sustained, or, on the contrary, the upper note ascends while the lowest note is sustained. As such exceptions began to appear in the literature, they became accepted by the theory.

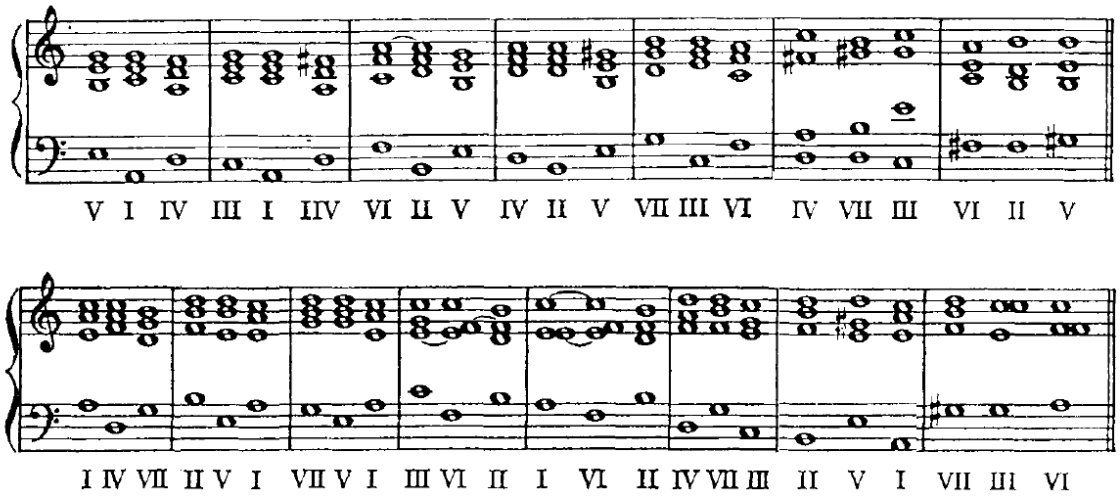


[**Figure 12.3**: Preparation and resolution of the seventh chords.]

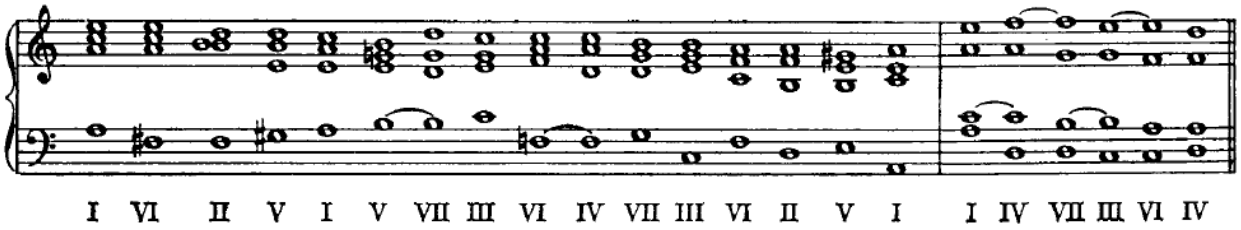
With the evolution of Western music, the preparation of seventh chords' dissonances has became quite optional. Monteverdi is one of the firsts that freely introduced unprepared dominant seventh

chords in its works. Later, Bach did the same with all the other seventh chords. Whenever the seventh is not prepared, it usually appears through a motion by scale degrees.

Figure 12.3 gives some examples of treatment of the seventh chords' dissonances. In figure 12.4, some examples in minor, and in the figure 12.5 two phrases in minor.



[**Figure 12.4**: Preparation and resolution of seventh chords in A minor. Taken from Arnold Schoenberg's *Theory of Harmony*.]



[**Figure 12.5**: Phrases in minor from Arnold Schoenberg's *Theory of Harmony*.]

Additional consonant or dissonant chords can be placed between the seventh chord and its resolution: in these cases, the seventh remains sustained in the same voice. It is also frequent that an inversion of the seventh chord precedes the resolution: here the seventh is taken by another voice, and resolves on a different octave.

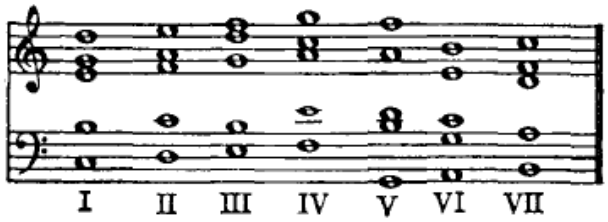
13-Ninth chords

The ninth chords are made up by placing another third above the seventh chords, so that a major or minor ninth interval with the bass is produced. Thus, they comprehend five notes: root, third, fifth, seventh and (major or minor) ninth.

The seven notes of the diatonic scale produce the following ninth chords (see also figure 13.1):

- **C**-E-G-B-D
- **D**-F-A-C-E
- **E**-G-B-D-F
- **F**-A-C-E-G
- **G**-B-D-F-A
- **A**-C-E-G-B
- **B**-D-F-A-C

The ninth chords on the third and seventh degrees involve a minor ninth interval between the root and the ninth, the others a major ninth interval.



[**Figure 13.1**: Diatonic ninth chords.]

The note lying a major ninth interval above the bass coincides with its 9th harmonic: therefore, the ninth chord on the dominant degree, G-B-D-F-A (where the first four notes form a dominant seventh chord), comprehends all the first 9 harmonics of the root. It is remarkable that the 9th harmonic of the root is the only harmonic that all the notes of a major triad have in common: it is also the 7th harmonic of the third and the 3rd harmonic of the fifth.

If the A harmonic minor scale is considered, the following new chords arise from the presence of G#:

- **A**-C-E-G#-B
- **C**-E-G#-B-D
- **E**-G#-B-D-F
- **F**-A-C-E-G#
- **G#**-B-D-F-A

and if the melodic minor scale is considered, the following new chords appear thanks to the presence of F# and G#:

- **B**-D-F#-A-C
- **D**-F#-A-C-E
- **E**-G#-B-D-F#
- **F#**-A-C-E-G#
- **G#**-B-D-F#-A

All the 7 seventh chords discussed in the previous chapter are ninth chords without root. In fact, all the ninth chords can be viewed in two ways:

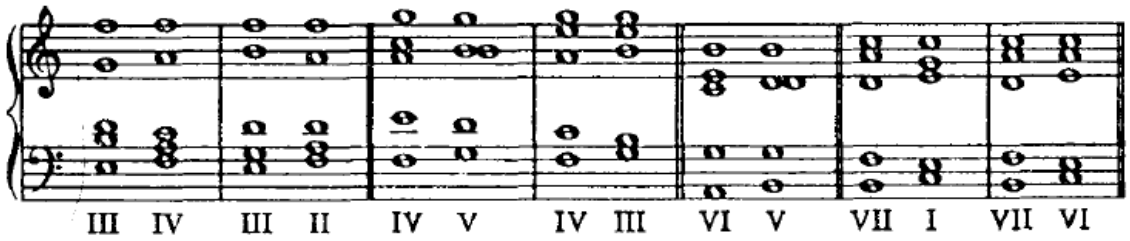
- Seventh chord + ninth
- Root + seventh chord

E.g., the ninth chord G-B-D-F-A can be interpreted as either the dominant seventh chord G-B-D-F plus a major third F-A, or the root G plus the half-diminished seventh chord B-D-F-A.

From this point of view, a second root can be assigned to each seventh chord: the omitted root of the corresponding ninth chord. E.g. the above half-diminished seventh chord B-D-F-A has two roots: B and G.

It is uncommon to use the inversions of the ninth chords because they sound really dissonant, though there are numbers of examples in the twentieth century's literature. Usually, they are inverted without the root, so that inversions of seventh chords are obtained.

In general, the addition of notes on the top of a chord doesn't have influence on the behavior of the other ones. Hence, the discussion in the previous chapter fully applies to the first 4 notes of the ninth chords in root position. Accordingly, ninth chords should resolve into a chord with the same root only if the seventh remains stationary, i.e. if the seventh is resolved later in another chord (having different root). Usually the ninth and the seventh of a ninth chord move one step downward, as shown in figure 13.2, or can be sustained if the bass ascends.



[**Figure 13.2**: Examples of ninth chord's resolutions.]

The omission of both the seventh and the third in the chords with the major ninth produces the second inversion of the **perfect-fourth chord**, a chord made up by adding two perfect fourth. E.g.,

the omission of E and B in the ninth chord C-E-G-B-D produces C-G-D, the second inversion of the perfect-fourth chord D-G-C.

This second inversion, C-G-D, where D is the highest note, is a quite pleasant chord: in fact, G is the 3rd harmonics of C, and D is the 3rd harmonics G. The lack of the third allows this chord to be interpreted either as a major or a minor chord, and, although the ninth is technically a dissonance, it often doesn't need preparation. This inversion is very frequently used in popular music.

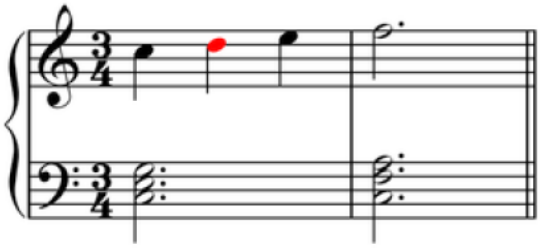
The second inversion of the perfect-fourth chord suggests that the perception of the major second interval considerably changes when an octave is added to it (to obtain a major ninth). In general, this doesn't apply to the other intervals, that don't be significantly influenced by the octave-extension. This exception can be explained by considering that the 9th harmonic lies much higher (more than three octaves) than its fundamental: consequently, the major ninth resembles the displacement of the harmonics more closely than the major second. Observe that, for the same reason, the presence of small intervals above the bass generally sounds bad, as noted in chapter 10.

The first inversion of the perfect-fourth chord D-G-C, i.e. G-C-D, includes a fourth (G-C) above the bass (which sounds dissonant) and a major second C-D: both these intervals are usually resolved through the descending motion of the fourth C to the third B of the major triad G-B-D (while the other two notes remain stationary). This is one of the most adopted dissonance's resolutions in Western music.

14-Embellishing notes

The introduction of passing notes between consecutive chords has been an ordinary practice since the early times of the development of harmony. These intervening notes don't take part to the leading melody and mostly assume an "ornamental" function. Therefore, they are also called *embellishing notes*, *non-chord tones*, *non-harmonic tones* or *figuration tones*.

In the traditional voice-leading, when a part has to make a leap e.g. of a third or a fourth interval, the embellishing notes occupy the remaining diatonic degrees between the interval's lower and upper notes. During this (ascending or a descending) motion by degrees, they form dissonant intervals with the other (stationary) parts, as in the example of the figure 14.1.



[**Figure 14.1**: Example of embellishing notes between consonant chords. The red note forms a dissonant interval with both the root and the third of I.]

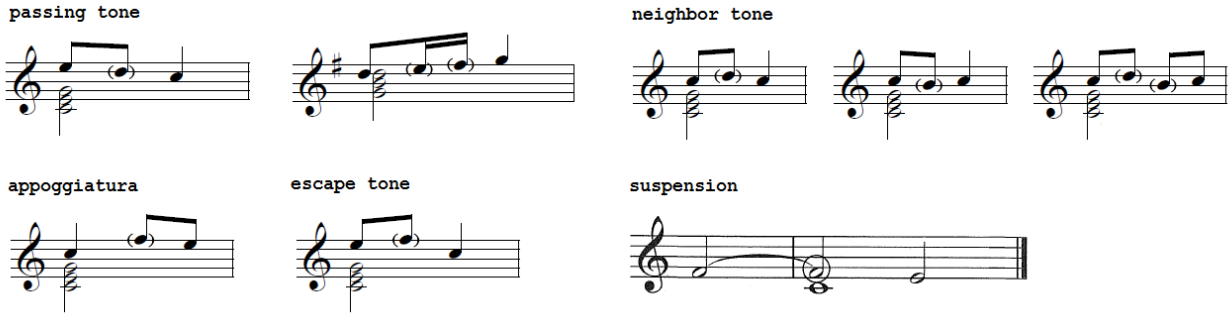
The degree to which an interval is perceived as dissonant also depends on how well its notes are integrated into the horizontal dimension of the music. From this point of view, the historical origin of the use of dissonant chords, in particular the seventh chords and the ninth chords, is frequently recognized in the incorporation of the embellishing notes into the chords that they melodically connect.

In fact, the introduction of passing notes is a valid method to produce harmony: when some notes are listened one after the other, the brain spontaneously evaluates the relationships between them, and perceives the chord that they would generate if they were sounded simultaneously. Whenever a chord is exclusively produced by a sequence of consecutively sounded notes, it is called **arpeggio**.

The embellishment notes provide a useful device for increasing the dynamics and the fluentness of the voice-leading. They are distributed in alternating voices and are usually positioned on the weak beats (or weak part of the beats). Among the most common usages, the following schemes have widely accepted names (see also figure 14.2):

- **Passing tone**: a passing note that fills a gap between two notes a third apart. Two adjacent passing tones can also be used to connect notes a fourth apart.
- **Neighbor tone**: a passing tone that occurs between two instances of the same note. It can also consist of two notes, making a *neighbor figure* (see figure 14.2).

- **Appoggiatura**: a passing note that is approached by leap and proceeds by step to the second chord. It is also called **incomplete neighbor tone**.
- **Escape tone** (or **echappée**): the opposite motion to that of the appoggiatura; that is, a passing note which is approached by step and proceeds by leap to the second chord. Also the escape tone is sometimes called "incomplete neighbor tone".
- **Suspension**: a particular case of resolution of the dissonance. It arises whenever the chord of resolution corresponds to the dissonant chord except for the dissonant note, that goes downward. That is, all the other notes remain sustained, and the dissonant note is the only one that moves.
- **Retardation**: an upward-resolving suspension.



[**Figure 14.2**: Typical use of embellishment notes.]

15-Tendency tones and cadences

As discussed in chapter 6, the sound of the dominant degree of the diatonic scale makes the tonic to be perceived as the missing fundamental frequency of a harmonic sequence. The dominant coincides (except for a small approximation) with the 3rd harmonic of the tonic, and this fact gives the dominant the tendency to converge to the tonic. That is, the tendency to fall a fifth below. The same applies to the corresponding diatonic triads: the transition from the dominant triad to the tonic triad, i.e. V-I, provides a strong sense of conclusion.

On the other hand, for the same reason (the dominant belongs to the harmonics of the tonic), also the opposite movement, from tonic to dominant, appears quite spontaneous, but to a lesser extent. This movement corresponds to the leap a fifth above.

Hence, in general, the preferred destinations where a note can go are the notes lying a fifth below and a fifth above. If the tonic is taken as destination, vice-versa, the notes that most spontaneously arrive to it will be the dominant and the subdominant degrees. Consequently, among the diatonic triads, V and IV provide the strongest connections to I. In particular, V-I is stronger than IV-I.

In addition to the contribution of the harmonics, there is also a melodic element that makes both the progressions V-I and IV-I particularly solid. Consider the two semitones of the diatonic scale: the first semitone lies between the third and the fourth degrees, the second is between the seventh degree (the leading-tone) and the tonic. Now, it is possible to observe that:

- Since the semitone is the half of a tone, the movements of the melody through the semitones are shorter, and thus they are more spontaneous than those through the tones.
- The tonic and the third degrees of the diatonic scale correspond respectively to the 1st and 5th harmonics of the tonic, while the fourth and the seventh degrees don't belong to the initial harmonics of the tonic.

Consequently, the seventh degree tends to move through the adjacent semitone toward the tonic, while the fourth degree tends to move through the adjacent semitone toward the third degree. The notes that behave like the diatonic scale's seventh and fourth degrees are called **tendency tone**s. The leading-tone is also called **ascending tendency tone**, and the fourth degree is also called **descending tendency tone**.

A tendency tone is a note lying a semitone away from a second note, and spontaneously needs to be followed by this second note: this movement is also called *resolution* of the tendency tone, since it is somewhat similar to the resolution of a dissonance. E.g., in C major, the tendency tones are B and F, and they tend to move respectively toward C and E.

The resolutions of the diatonic scale's tendency tones take part to the connections V-I and IV-I. To be specific, consider e.g. C major: when connecting V (G-B-D) to I (C-E-G), B resolves to C; when connecting IV (F-A-C) to I, F resolves to E.

In the case of minor tonalities, the leading-tone (in A minor, G#) is supplied by the melodic and harmonic minor scales. Thanks to G#, the A minor's dominant triad becomes major: thus, in the progression from V (E-G#-B) to I (A-C-E), G# goes to the tonic A. The descending tendency tone of A minor coincides to that of C major (that is, F): in the connection from IV (D-F-A, a minor triad) to I, F resolves going downward to the fifth of the tonic triad. Observe that F is the sixth degree of the A natural minor scale: as discussed in chapter 11, the natural minor scale is especially used in descending melodic motions. This is coherent with the tendency of F to move downward toward E.

Both in major and minor keys, the progression V-I from the (major) dominant triad to the tonic triad is called **authentic cadence** or **perfect cadence**. This connection is extremely solid, and the motion to the tonic triad is associated to a strong sense of conclusion, especially when the triads are in root position and the leading-tone is assigned to the highest part.

When placing an authentic cadence in the music, the minor seventh is often added to V, producing a dominant seventh chord (in C major, G-B-D-F; in A minor, E-G#-B-D). Thanks to this additional dissonant element, the cadence is reinforced by the resolution of the seventh interval, and, in major key, also by the resolution of the descending tendency tone (F, the seventh itself).

Every connection to the tonic triad that involves the resolution of the leading-tone can be considered a variant of the authentic cadence. In particular, two of the possible alternatives to V-I are the following:

- VII-I, both in major and minor keys. VII is the diminished triad on the leading-tone (in C major, B-D-F; in A minor, G#-B-D). This triad can be obtained by omitting the root of the dominant seventh chord on the dominant degree. As discussed in chapter 12, the omitted root can be thought as a second root for VII.
  The minor seventh or the diminished seventh is often placed upon VII. In major keys, the minor seventh produces a half-diminished seventh chord (in C major, B-D-F-A); in minor keys, the diminished seventh produces a diminished seventh chord (in A minor, G#-B-D-F). In both the cases, both the tendency tones find their resolution when going to the tonic triad.
- III-I, both in major and minor keys. In minor, III is the augmented triad (in A minor, C-E-G#).

Examples of the authentic cadence are shown in figure 15.1:

The transition from the subdominant triad to the tonic triad (IV-I) is called **plagal cadence** (shown in figure 15.2). It is a more gentle and solemn progression than the authentic cadence. Christian church hymns end with the word "Amen", which is almost always sustained by the plagal cadence. The descending tendency tone is involved in this cadence. As noted above, in the major keys the

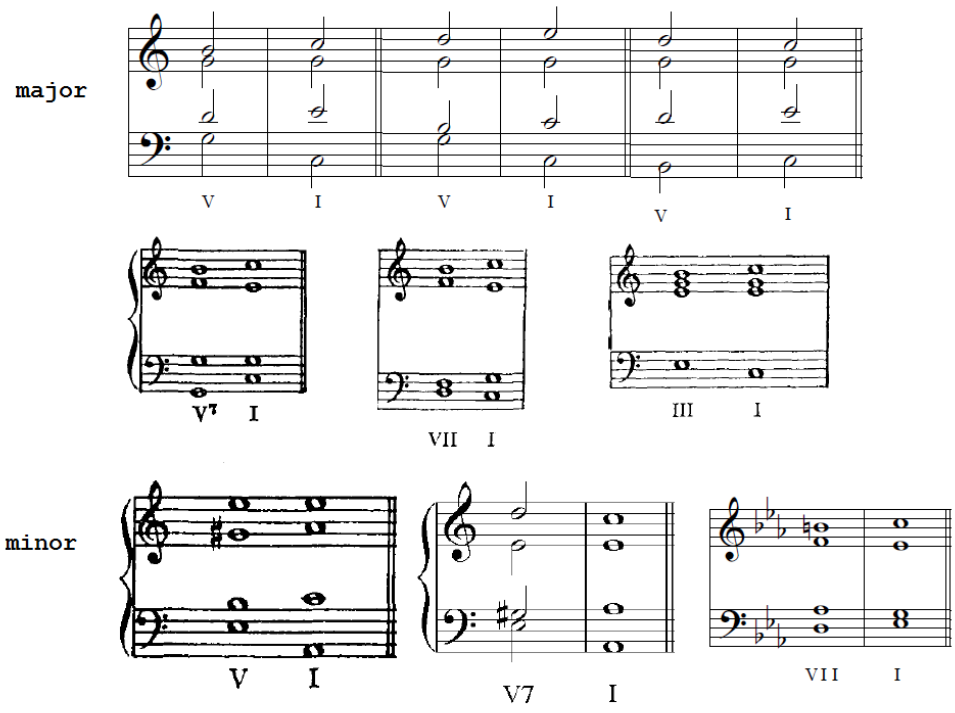
fourth degree resolves by moving to the third of the tonic triad, whereas in the case of minor keys the sixth degree resolves to the fifth. Observe that, in minor tonalities, IV is not affected by the accidental on the sixth degree that characterizes the melodic minor scale, and therefore it remains a minor triad.

Possible variants of the authentic cadence are the following:

- II-I, both in major and minor keys. In C major, D-F-A connects to C-E-G; in A minor, B-D-F connects to A-C-E.
- VI-I, in minor keys. In A minor, F-A-C connects to A-C-E.

Examples of the plagal cadence are shown in figure 15.2:

When using the authentic cadence, the subdominant triad is frequently placed before V, producing IV-V-I. This is one of the strongest chord progressions in tonal music. In minor keys, particular carefulness is needed when connecting the (minor) subdominant triad to the (major) dominant triad: an augmented second can be produced between the (unraised) sixth degree (the third of IV) and the chromatically raised seventh degree (the third of V).



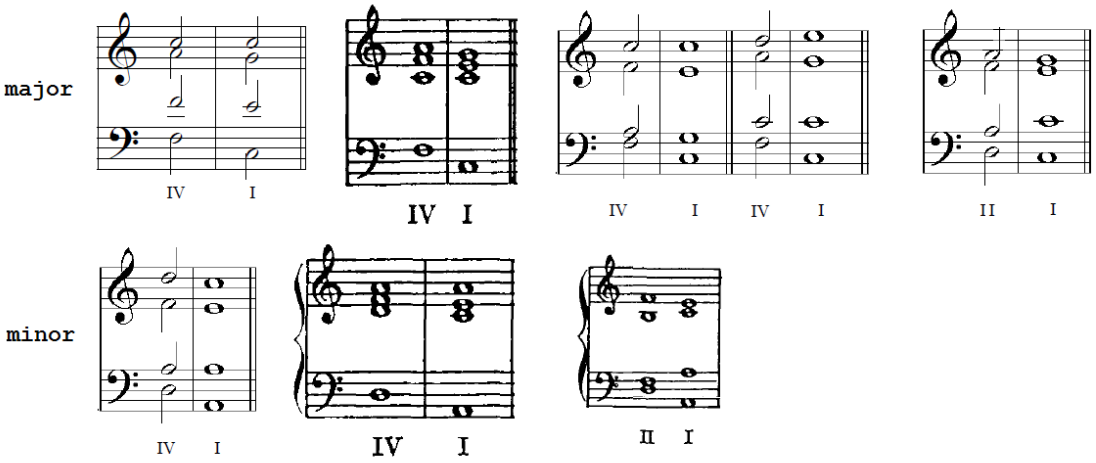
[**Figure 15.1**: Authentic cadences.]

A large amount of common practice music ends through the authentic cadence V-I. Whenever the chord following V (or VII, or III) is not I, which is the chord the listener was expecting, the resulting cadence is called **deceptive cadence** or **irregular resolution** (shown in figure 15.3). Typical deceptive cadences are V-VI and V-IV (or VII-VI and VII-IV).

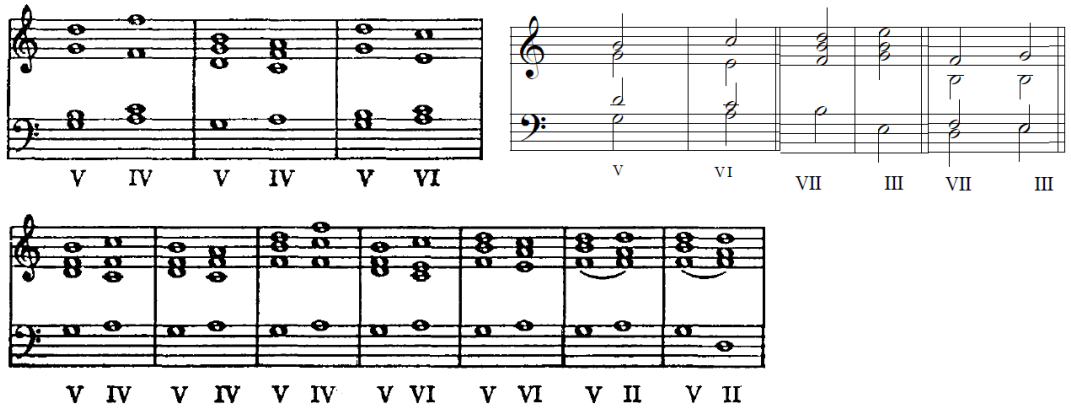
Since the melodic power of the tendency tones is provided by the presence of a semitone between them and their resolution, every chromatically altered note (lying a semitone far from a diatonic note) behaves like a tendency tone, as it will be discussed in the next chapter.

Coherently, the traditional voice-leading (see chapter 11) treats the chromatically raised notes as they were the (chromatically raised) leading-tone of the melodic and harmonic minor scales, and treats the chromatically lowered notes as they were the sixth degree of the natural minor scale (the descending tendency tone). Thus, chromatically raised notes tend to move a half-step upward, and chromatically lowered notes tend to move a half-step downward.

As a consequence, the notes that form a diminished interval (where the upper note is chromatically lowered, or the lower note is chromatically raised) have the tendency to approach one another, and the notes forming augmented intervals (where the upper note is chromatically raised, or the lower note is chromatically lowered) have the opposite tendency to diverge.



[**Figure 15.2**: Plagal cadences.]



[**Figure 15.3**: Deceptive cadences.]

16-Modulations

In tonal music, a modulation is the introduction of a new tonality. That is, the listener perceives the new tonic and the corresponding (major or minor) tonic triad as the most stable and conclusive sound. When a new tonality is fully adopted, usually a considerable change takes place in the music, and everything happens in the new key is emphasized. It is very common to use modulations for introducing new themes (musical phrases), or the same theme can have both a major-key version and a minor-key version.

However, a modulation doesn't have to necessary be a complete and permanent transition to a new key. It can be a temporary event, so that after few chords the original tonic spreads his influence again. Two tonalities can also cohabit together: the tonic of the new tonality may not prevail over the old one, and the notes of both the keys are used. Whenever two or more tonalities continue to be simultaneously present, and in general when the tonality remains ambiguous, the term **suspended tonality** is used. This concept will be discussed in chapter 19.

The transition from one tonality to another usually doesn't happen all at once. An unexpected modulation must have specific artistic purposes: whenever the notes which are typical of the new key don't be introduced in the music through a phase of transition, the change of key is generally weak and undecided. The listener has to lose the reference sounds of the tonality that he is leaving, the attraction of the old tonal center should progressively become weaker, providing the conditions for a new tonal center to be established.

The preparation of the new key can include a variety of additional elements, such as embellishment tones and/or particular rhythmical patterns that creatively bring the listener toward the new tonality. These solutions, anyway, reflect (to some extent) the same principles in which a simple sequence of chords finds its foundations. The most general way to illustrate the means for obtaining good modulations, therefore, consists of considering sequences of bare chords only.

Since the music has to gradually take the distances from the old tonic, the adoption of a new tonality (i.e. a new diatonic scale) generally becomes harder as the number of notes that have to change increases, that is, as the distance in the circle of the fifths (see chapter 6) between the old and the new keys augments. However, the bare difference between the key signatures is not the only element to be considered: e.g., the modulation between parallel tonalities is facilitated by the presence of a common dominant triad, as discussed later.

Once the influence of the old tonic has been sufficiently weakened, an energetic chord progression should establish the new tonic triad: the most incisive chord progressions to the tonic triad are the authentic and the plagal cadences, or their variants, that were discussed in previous chapter. In particular, the authentic cadence is the strongest and most used device for clearly introducing the new tonic triad. Although the use of these cadences is the standard practice, a modulation doesn't absolutely have to go to the new tonic triad. If only the other diatonic chords of the new tonality

appear, the resulting modulation is somewhat "hidden": the new key enters without explicitly manifesting its most important element, the tonic triad. Here a deceptive cadence may be used.

The melodic strength of the authentic and the plagal cadences is given by the resolutions of the two tendency tones. The leading-tone (ascending tendency tone) moves to the tonic, and the diatonic scale's fourth degree (descending tendency tone) moves either to the third (in major) or to the fifth (in minor) of the tonic triad.

Looking at the circle of the fifths, the tendency tones are the notes that change when going from a major tonality to either the next or the previous one in the circle. E.g., when going from C major to G major, a sharp on F is added: F is the descending tendency tone of C major, and the accidental produces the ascending tendency tone (F#) of G major. Going in the opposite direction, from C major to F major, a flat on B is added: B is the ascending tendency tone of C major, and the accidental produces the descending tendency tone (Bb) of F major, which is also the descending tendency tone of its relative minor (D minor).

Thus, in general, when going around the circle of the fifths in the direction where the number of sharps increases, the sharp that is added upon the descending tendency tone of a major tonality produces the ascending tendency tone (the leading-tone) of the next major tonality. Vice-versa, when going in the direction where the number of flat increases, the flat that is added upon the ascending tendency tone of a major tonality produces the descending tendency tone of the next major tonality (and its relative minor). The tendency tones drive the circular relationship: they lead from one tonality to another, being the notes that change in the transition from one key to the adjacent ones.

As a consequence, considering a diatonic scale, the ascending tendency tones of the major tonalities with more sharps or fewer flats are chromatically raised notes, and the descending tendency tones of the major and minor tonalities with more flats or fewer sharps are chromatically lowered notes.

This is coherent with the fact that chromatically raised notes tend to behave like a leading-tone, and chromatically lowered notes tend to behave like the descending tendency tone.

Furthermore, considering a diatonic scale, the descending tendency tones of the 6 closest major tonalities with more sharps or fewer flats don't be chromatically altered notes, and the ascending tendency tones of the 6 closest major tonalities with more flats or fewer sharps don't be chromatically altered notes.

To be specific, when e.g. in C major (A minor) tonality:

- The ascending tendency tones of the major tonalities with more sharps, i.e. G, D, A, E, B, F#, C# major keys, are chromatically raised notes. They are respectively F#, C#, G#, D#, A#, E# and B#.

- The descending tendency tones of the major tonalities with more flats, i.e. F, Bb, Eb, Ab, Db, Gb, Cb major keys, are chromatically lowered notes. They are respectively Bb, Eb, Ab, Db, Gb, Cb and Fb.
- The ascending tendency tones of the 6 closest major tonalities with more flats, i.e. F, Bb, Eb, Ab, Db, Gb major keys, don't be chromatically altered notes. They are respectively E, A, D, G, C, and F.
- The descending tendency tones of the 6 closest major tonalities with more sharps, i.e. G, D, A, E, B, F# major keys, don't be chromatically altered notes. They are respectively C, G, D, A, E and B.

In the case of minor tonalities, their descending tendency tone is the same note of the relative major (the sixth degree of the natural minor scale), and therefore the above discussion applies. The ascending tendency tone, on the contrary, is the artificial leading-tone. Thus, when in C major (A minor) tonality:

- The ascending tendency tones of the minor tonalities with more sharps, i.e. E, B, F#, C#, G#, D#, A# minor keys, are chromatically raised notes. They are respectively D#, A#, E#, B#, F##, C## and G##.
- The ascending tendency tones of the minor tonalities with more flats, i.e. D, G, C, F, Bb, Eb, Ab minor keys, are respectively C#, F#, B, E, A, D, G, where the lasts don't be chromatically raised notes (but they are chromatically raised notes in their own tonalities, where a natural sign cancels the corresponding flat in the key signature).

In the music of the common practice period the typical way for modulating to a new tonality consists of the introduction, at the end of an appropriate phase of transition, of the tendency tone that corresponds to a chromatically altered note in the old key. This tendency tone usually takes part to an authentic cadence (V-I or VII-I) that falls to the new tonic triad.

To be specific, if the authentic cadence is used, it is possible to distinguish the following cases:

- The modulation brings to a major or a minor tonality, and the leading-tone of the new tonality is a chromatically altered note in the old tonality. In this case, the leading-tone can take part to the authentic cadence either as the (major) third of the dominant triad or as the root of VII.
- The modulation brings to a major tonality, and the descending tendency tone of the new tonality is an altered note in the old tonality. In this case, this tendency tone can take part to the authentic cadence as the seventh of the dominant chord (the seventh above the dominant degree is the fourth degree) and resolves by going to the third of the tonic triad.
- The modulation brings to a minor tonality, and the descending tendency tone of the new tonality is the only tendency tone being an altered note in the old tonality (e.g., when modulating from C major to C minor or F minor). In this case, this tendency tone cannot take part to the authentic cadence as the seventh of the dominant, as it

happened in the previous case (in minor keys, the seventh of the dominant doesn't be the minor scale's sixth degree), but it can participate as the minor ninth of the dominant chord, that resolves by going to the fifth of the tonic triad. However, frequently the cadence remains lacking of a chromatically altered note being a tendency tone.

For example, when modulating from C major or A minor to G major or G minor, the new keys' ascending tendency tone, F#, is the third of the new key's dominant triad D-F#-A, that connects to the new tonic triad G-B-D or G-Bb-D. When modulating to F major, the new key's descending tendency tone, Bb, is the seventh of the new key's dominant triad, i.e. C-E-G-Bb, that connects to the tonic triad F-A-C. When modulating to F minor, the new key's descending tendency tone, Db, is the minor ninth of the new key's dominant triad: C-E-G-Bb-Db connects to F-Ab-C.

In the old key, the major dominant triads that are used for modulating through an authentic cadence (with or without seventh, or ninth) are called **secondary dominant**s, and the leading-tones that are involved are called **secondary leading-tone**s. The secondary dominants bring the music toward the (major or minor) tonality where they are dominants (in minor, artificial major dominants). In the common practice music, the majority of the modulations exploit the secondary dominants of the old key.

The authentic cadence can also be produced by adopting, in place of the dominant major triad, the diminished triad on the leading-tone. The diminished triads whose root is a leading-tone are also called **leading-tone chord**s. As in the case of the dominant triad, placing these triads in the old key allows modulating to the (major or minor) tonalities where they are leading-tone chords (in minor, artificial leading-tone chords).

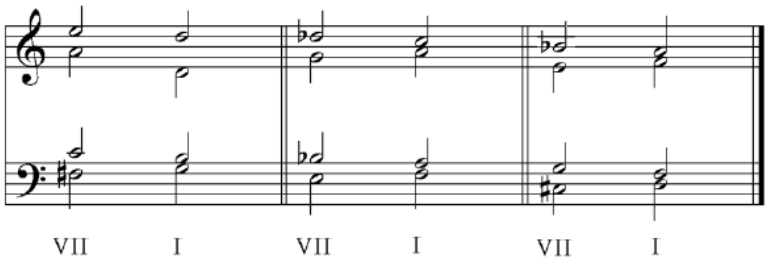
The seventh is usually added to the leading-tone triads. This seventh can be either the minor seventh (so that a half-diminished seventh chord is obtained) or the diminished seventh (so that a diminished seventh chord is obtained). When modulating to a major key, both the minor seventh and the diminished seventh are possible, but when modulating to a minor key, only the diminished seventh is to be used because it is the descending tendency tone. The three cases are shown in figure 16.1.

E.g., when F#-A-C is used for modulating to G major, both E and Eb can be the seventh; when F#-A-C is used for modulating to G minor, then Eb will be the seventh because it is the descending tendency tone of G minor. In both the cases, the additional seventh goes downward to the fifth D of the tonic triad.

As noted above, although the authentic cadence works very well for reaching a tonic triad, placing an unexpected cadence in the middle of the music rarely produces good modulations. The cadence is only the final step of the transition from the old key to the new key, that is, it should be preceded by a progression of chords that weakens the tonic and introduces the typical notes of the new key. Within such progression the secondary dominants and the leading-tone chords can be used as well. For example, they can take part to a deceptive cadence (the new tonic triad doesn't appear

after them), or a weaker form of the authentic cadence may appear, where e.g. the new tonic triad is in inversion or it includes a dissonant note.

In general, the notes that are distinctive of the new key (those that change in the key signature and, eventually, the artificial leading-tones of minor keys) can be used to produce dissonant chords or the consonant triads of the new tonality. These chords will considerably help to bring the music far from the old tonic, anticipating the appearance of the modulating cadence.



[**Figure 16.1**: Examples of authentic cadence involving leading-tone chords with added seventh. When modulating to a major key both the half-diminished seventh chord and the diminished seventh chord can be used. When modulating to a minor key (last example), only the diminished seventh chord has to be adopted because the seventh is the descending tendency tone.]

In the case of modulations to the closest tonalities in the circle of the fifths, e.g. from C major to A minor, G major, E minor, F major and D minor, the weakening of the old tonic can be obtained through the use of the triads that the old and the new keys have in common. These triads are sometimes called *pivot chords*, and can be interpreted as belonging to both the initial and final keys.

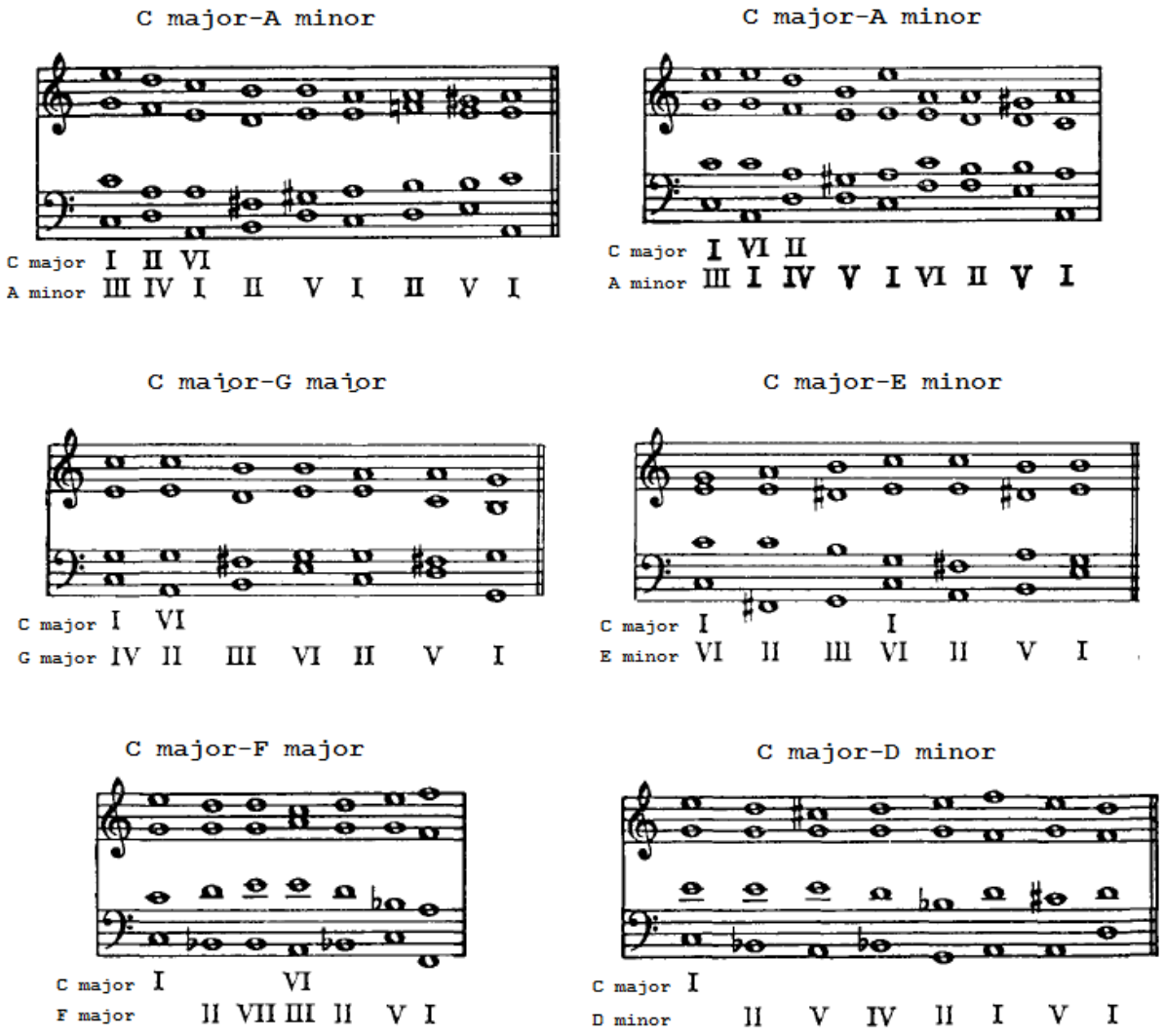
The modulation from C major to A minor is the simplest, since the diatonic scale is the same and the two keys have all the diatonic triads in common. Whereas unprepared modulations to more distant keys usually provide abrupt and unexpected changes in the music, in the case of modulations to the relative minor (or the opposite modulation to the relative major), on the contrary, the transition is so spontaneous that the presence of an authentic cadence frequently gives enough emphasis for the modulation to be well perceived. When modulating to the relative minor, it is advisable to anticipate the change of key by introducing the chromatically raised notes of the melodic minor scale, and avoiding the diatonic triads the comprehend the unaltered seventh degree (in major, I, III and V). Two examples of modulation from C major to the relative minor are shown in figure 16.2.

The modulations from C major to the adjacent keys in the circle of the fifths (G major, F major, and their relative minor keys) require a bit of carefulness: the modulating cadence should be anticipated by a sequence (eventually very short) of chords that avoids the unaltered version of the only note that changes in the key signature. E.g., when modulating from C major to G major, the diatonic triads involving F (II, IV and VII of C major) shouldn't precede the final cadence, and when

modulating to F major the diatonic triads involving B (III, V and VII of C major) shouldn't precede the cadence.

Moreover, when modulating to E minor or to D minor, their raised seventh degree (artificial leading-tone) should appear before the cadence, but not their raised sixth degree: in fact, the unraised sixth degree is their descending tendency tone. Observe that, on the contrary, in the modulation to the relative A minor the sixth degree F can either be altered or not.

In figure 16.2 some examples are given. It can be noted that when modulating to E minor and D minor two chromatically altered notes are used for introducing the new key: the artificial leading-tone and the note that changes in the key signature. In the case of the modulations to G and F major, only one altered note is available.

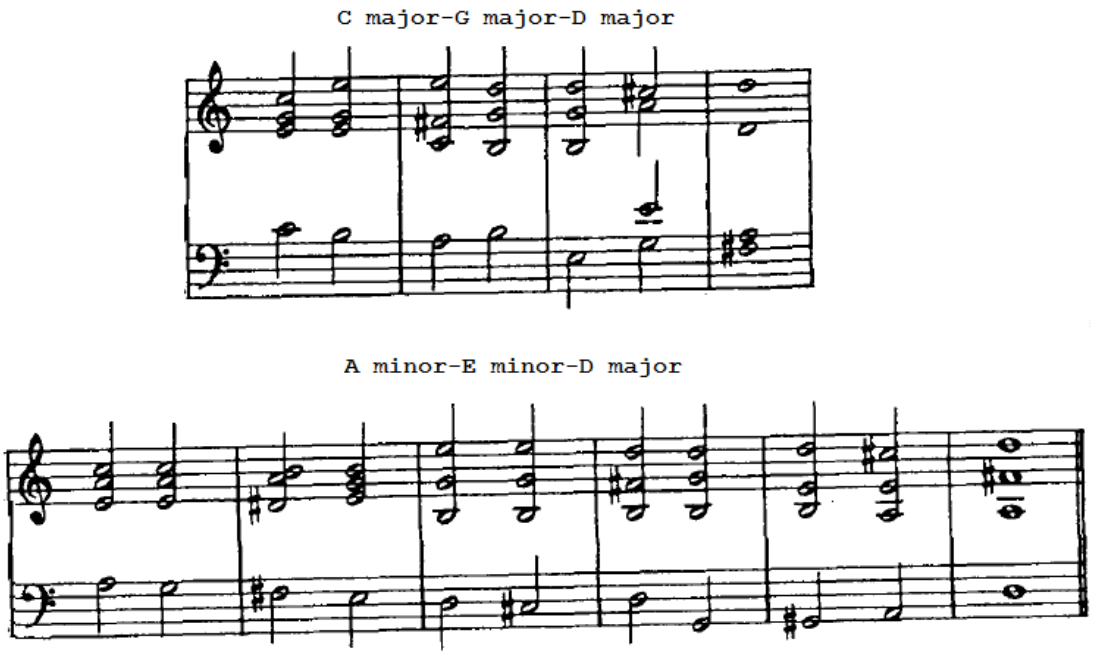


[**Figure 16.2**: Examples of modulation to the relative major/minor key and to the adjacent tonalities in the circle of the fifths. The penultimate chord is a secondary dominant. Taken from Arnold Schoenberg's *Theory of Harmony*.]

In the case of modulations between tonalities that are more distant in the circle of the fifths (the key signatures differ in more than one note), the scarcity of common notes requires to use more elaborated solutions for building the progression that precedes the final cadence.

One of the possibilities is the use of intermediate keys, i.e. "passing" or "intervening" tonalities that lie between the old tonality and the new tonality in the circle of the fifths, so that the whole modulation is subdivided into two or more consecutive modulations to adjacent tonalities. E.g., for modulating from C major or A minor to D major or B minor, the intermediate key will be G major or E minor; for modulating from C major or A minor to Bb major or G minor, the intermediate key will be F major or D minor.

he figure 16.3 shows modulations from C major and A minor to D major.



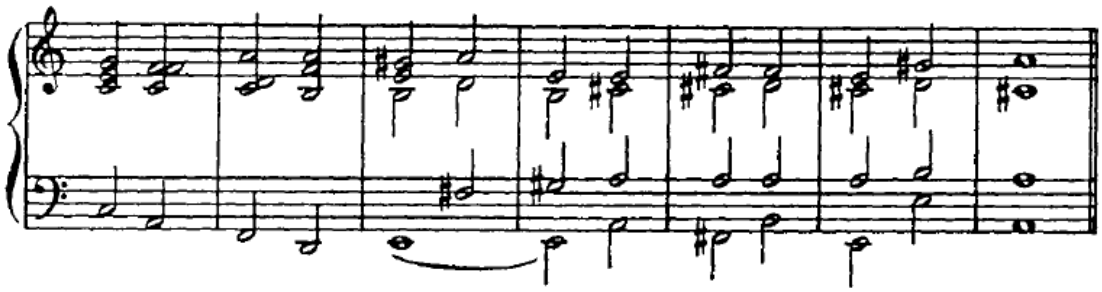
[**Figure 16.3**: Modulation through intermediate keys. Taken from Arnold Schoenberg's *Theory of Harmony*.]

When modulating to the keys that differ in three notes, e.g. from C major or A minor to A major, F# minor, Eb major and C minor, it is possible to exploit the fact that in minor the authentic cadence involves a major dominant triad: this triad is the same dominant triad of the parallel major key, and therefore it can be interpreted as a member of both the two tonalities. The common dominant triad allows a direct modulation between them. E.g., for modulating from C major to C minor, or, vice-versa, from C minor to C major, the same dominant triad G-B-D is involved in the cadence.

The sound of the dominant chord should be sustained long enough for its derivation from the old key to be forgotten; then the authentic cadence can immediately follow. Since keeping immobile the dominant triad rarely produces good results, usually only its root (the dominant degree of both

the keys) remains stationary, while the other parts proceed with suitable chords until the final whole triad. These transitional chords can be dissonant with the sustained dominant.

The possibility of a short connection between parallel tonalities can be exploited for modulating to the relative major/minor of the parallel minor/major key: when going from C major to Eb major, C minor can appear as intermediate key; when going from C major to A major, A minor can appear as intermediate key. Moreover, when going from C major to F# minor, both A minor and A major can appear as intermediate keys. Similar considerations apply to the same modulations from A minor. E.g., when modulating from A minor to Eb major, both C major and C minor can be placed in the middle.



[**Figure 16.4**: Example of modulation from C major, to A minor, then to A major. The dominant triad E-G#-B of the parallel keys A minor and A major is used for modulating from the first to the second (third and fourth measures). Taken from Arnold Schoenberg's *Theory of Harmony*.]

The bridge that is created by the parallel tonalities also allows to proceed further to the keys that differ in four notes. E.g., for modulating from C major to E major it is possible to transit through A minor and A major, the last being adjacent to E major in the circle of the fifths.

The modulations that have been discussed make use of the most traditional methods of the harmonic practice. The common chords offer the possibility to join distant keys in the circle of the fifth through the insertion of intermediate keys, but when this strategy is adopted the length of the transition between the initial and the final key increases as the number of required intermediate keys augments. Moreover, the voice-leading has been entirely non-chromatic: the altered notes were treated as they were the seventh or the sixth degrees of the minor scales, as discussed in chapters 11 and 15.

There exist stronger means for composing modulations. As it will be discussed in chapters 18 and 19, the chromatic voice-leading and the reinterpretation of the chords can be exploited to produce direct connections between any couple of chords and keys.

17-Diminished seventh chord and augmented triad

The rewriting of a chromatically altered note by using a different accidental (e.g., to write Eb in place of D#, or E# in place of F) is called **enharmonic change** or **enharmonic reinterpretation**. Enharmonic changes in the notation can have both melodic and harmonic purposes. When the music allows a chord to be interpreted as belonging to more than one tonality, the enharmonic change of the notes consents to make explicit his membership to a tonality (a key signature) that is different from the one that is adopted, and allows to indicate that the melody of the parts is following the profile of a different scale.

For example, consider the augmented triad C-E-G#, which is most naturally obtained in A minor (see chapter 7). If G# is enharmonically changed to Ab, then the major third interval E-G# becomes the diminished fourth E-Ab. Now, since the inversion of E-Ab is the augmented fifth Ab-E, the enharmonic change alters the root of the triad: C-E-G# is transformed into the first inversion of the augmented triad Ab-C-E, which is most naturally obtained in F minor.

Among the chords that are most inclined to be considered elements belonging to many tonalities, and therefore that are most often enharmonically rewritten, the diminished seventh chord and the augmented triad are very important examples. Observe that their structure is very special: the diminished seventh chord is made up by joining three minor thirds together, and the augmented triad is made up by joining two major thirds. Consequently, the diminished seventh chord divides the octave in four equal parts (the sum of four minor thirds is equal to 12 semitones), and the augmented triad divides the octave in three equal parts (the sum of three major thirds is equal to 12 semitones as well).

E.g., consider the diminished seventh chord B-D-F-Ab. All its notes lie a minor third distant one from each other, and if another minor third was placed above Ab, Cb would be obtained: through enharmonic change, Cb can be written as another B. Similarly, by adding a major third on the top of the augmented triad C-E-G#, B# is obtained, which can be rewritten as another C.

As a result of these singular facts, each of the inversions of a diminished seventh chord is another diminished seventh chord, and each of the inversions of an augmented triad is another augmented triad. Therefore, by exploiting all the 12 semitones of the octave, it is possible to write only three different diminished seventh chords, and only four different augmented triads (shown in figure 17.1). To be specific, taking e.g. the first three degrees of the chromatic scale, C, C# and D, all the available diminished seventh chords are the following:

- C-Eb-Gb-Bbb
- C#-E-G-Bb
- D-F-Ab-Cb

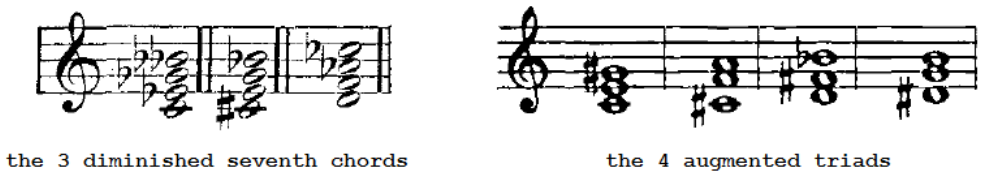
All the other diminished seventh chords which are built on the remaining chromatic scale's degrees coincide with the inversions of one of the above three (by enharmonic changing some notes).

Similarly, taking the first four degrees of the chromatic scale, all the available augmented triads are:

- C-E-G#
- Db-F-A
- D-F#-A#
- Eb-G-B

and those which are built on the remaining degrees coincide with the inversions of one of the above ones (by enharmonic changing some notes).

Because of the symmetric nature of the augmented triad and the diminished seventh chord, their root loses all its most important attributes. In fact, whenever the bass is changed, the amount of harmonics that the upper notes have in common with it remains exactly the same.



[**Figure 17.1**: The three diminished seventh chords and the four augmented triads.]

The diminished seventh chord is most naturally derived as the seventh chord on the seventh degree of the harmonic minor scale (see chapter 12). But the enharmonic change of its notes allows to consider it member of many minor tonalities. E.g., the diminished seventh chord G#-B-D-F can be written as:

- G#-B-D-F: it belongs to A minor.
- G#-B-D-E#: it belongs to F# minor.
- G#-B-C##-E#: it belongs to D# minor.
- Ab-B-D-F: it belongs to C minor.
- Ab-Cb-D-F: it belongs to Eb minor.
- Ab-Cb-Ebb-F: it belongs to Gb minor.

Moreover, one diminished seventh chord can be interpreted as four different ninth chords without root. In fact, four different basses (roots) can be respectively assigned to its root position and to the three inversions, producing four different ninth chords. To be specific, taking e.g. the diminished seventh chord F#-A-C-Eb, the following ninth chords can be produced:

- **D**-F#-A-C-Eb
- **F**-A-C-Eb-Gb
- **Ab**-C-Eb-Gb-Bbb
- **B**-D#-F#-A-C

where some notes of the diminished seventh chord have been enharmonically changed to preserve the nature of the ninth chords' intervals. Observe that the four basses D, F, Ab and B are the members of another diminished seventh chord.

The diminished seventh chord is typically used for expressing passion, tension or other strong feelings. Since it can easily be interpreted as an element of various tonalities, it can be connected with almost any other chord without difficulties, and can be introduced without preparing its dissonances, as it will be discussed in the next chapter.

Analogous considerations allow to see also the augmented triad from at least three different tonalities:

- C-E-G#: it belongs to A minor.
- B#-E-G#: it belongs to C# minor.
- C-E-Ab: it belongs to F minor.

Observe that the fifths of the above triads, G#, B# and E (which, in F minor, is written by placing a natural sign), all are chromatically raised tones.

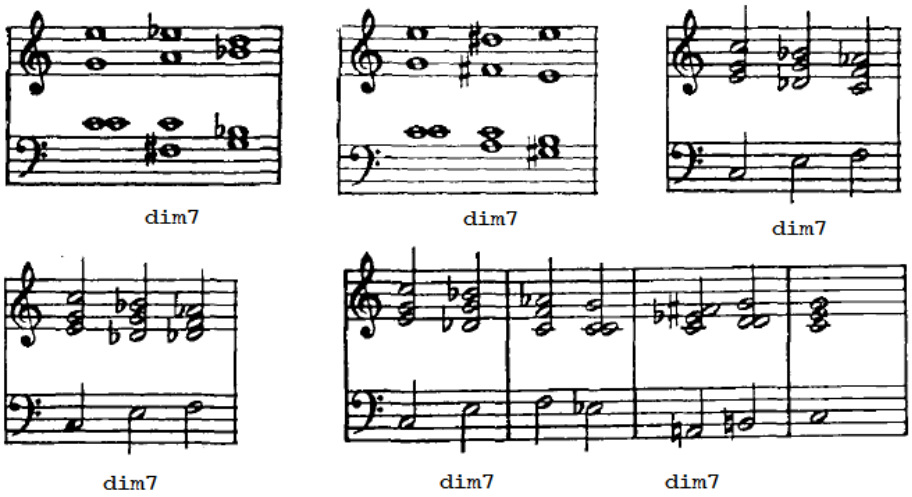
18-Chromatic voice-leading and vagrant chords

In the previous chapter the diminished seventh chord and the augmented triad have been analyzed in terms of their predisposition to be interpreted as belonging to many tonalities. When the music includes a chord that can be considered member of different keys, and it doesn't have a clear relationship with the major or minor scale that is used, this chord is called **vagrant chord**, a term coined by Schoenberg. Although the diminished seventh chord and the augmented triad are very noticeable examples, especially the first, all the chords can be used as vagrant chords.

The possibility for a chord to be vagrant is given by the context in which it appears: whenever a chord can be placed between two chords that don't belong to same key, the music approaches it as it belonged to one key, and quits it as it belonged to another key. So, this chord acquires "multiple meanings", and can be considered a vagrant chord.

Thus, the chord's membership to many tonalities strongly depends on the possibility of *directly* joining it with chords that don't be recognizable as elements of the same key. For such connections to be pleasant, the notes have to move mostly by semitones (or to remain stationary): half-step motions allow to melodically go from a diatonic note to a non-diatonic note, and this ability can be exploited to build smooth connections between any couple of chords.

In other words, the emancipation of a chord from the tonality is related to the possibility of connecting it to the previous and to next ones through chromatic voice-leading: a diatonic note can be followed by a chromatically altered note. Here the melodic strength of the semitones, discussed in chapter 15, plays a primary role.



[**Figure 18.1**: Examples of use of the diminished seventh chord as vagrant chord.]

The use of chromatic voice-leading can be thought as a consequence of the adoption of the chromatic scale as a basis for the tonality. Whereas in the traditional voice-leading the motion of the parts proceeds through segments of either the diatonic scale or the melodic minor scale, the

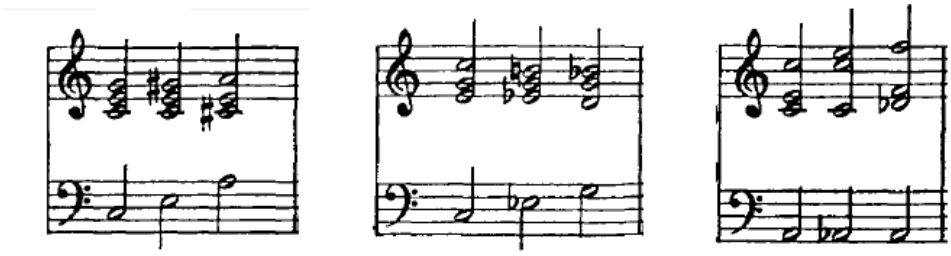
chromatic voice-leading always permits the parts to move from each chromatic scale's degree to the adjacent ones. After all, both the diatonic and the chromatic scale are themselves melodies.

Consequently, if two consecutive chords respectively comprehend a note and its chromatic alteration, then the same part should move from the unaltered to the altered note. If it doesn't happen, the consecutive presence of the unaltered and the altered notes in different parts is called **false relation** or **cross relation**. Avoiding false relations consents the parts to move through portions of the chromatic scale. This rule can be neglected when the notes could be enharmonically changed, since the ear is ready to give the altered note different harmonic meanings.

The enharmonic rewriting of the notes can help to show the derivation of a chord that appears in a context where it can have multiple interpretations. When writing the accidentals, it is also desirable to be guided by the key which the passage most resembles. Anyway, since the score has to be quickly read by the performer, the best notation has the fewest accidentals.

The indefinite character of vagrant chords is usually adopted for mitigating chord connections that otherwise would be forced (and would sound harsh). Since they "wanders away" from the tonal center, they provide the shortest way for introducing another tonality. Thus they are powerful devices for making modulations, as will be discussed in the next chapter.

The vagrant chords' membership to different tonalities is to some extent favored if they are dissonant chords: while the harmonics involved in the sound of the major and minor triads make them good representatives of the corresponding major and natural minor scales, in the case of dissonant chords the harmonics' spectrum presents greater variety. Many non-diatonic notes appear among the loudest (and/or most numerous) harmonics, weakening the bond to a particular key and supporting the connections to a greater range of chords.



[**Figure 18.2**: Examples of augmented triad's usage as vagrant chord.]

For example, the harmonics of the diminished seventh chord comprehend all the 12 notes: consequently, this chord has a conspicuous amount of harmonics in common with any one of the possible 24 major and minor triads. The diminished seventh chord is indeed one of the chords that are most frequently used as vagrant, and it is one of the firsts to have been historically used for connecting "unrelated" chords. It have been adopted in modulations since the times of Bach. The chromatic voice-leading allows the two interlocking tritones to move by half-steps in contrary or similar motion: when the tritones are augmented fourths, they would resolve outward to sixths; if the tritones are diminished fifths, then move inward to thirds.

As discussed in the previous chapter, also the augmented triad has a noticeable predisposition to be used for obtaining wandering harmonies. See for example the figure 18.2. It is very common to prepare the augmented triad with the corresponding unaltered form, i.e. the major triad on the same root. In this case, the fifth is the only note that changes: such movement emphasizes its historical origin as embellishment tone (see chapter 14).

The introductions of non-diatonic notes in the tonality, and the possibility of reinterpreting the chords in different keys, produce additional elements to the set of chords that have been already discussed.

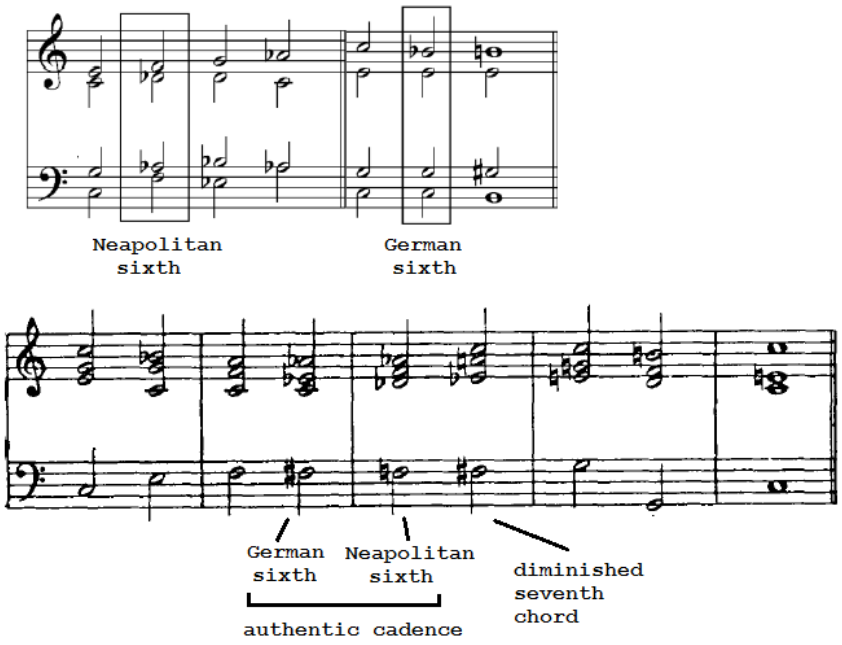
The **Neapolitan sixth** is the use of a non-diatonic major triad as a vagrant chord. The easiest way to obtain a non-diatonic major triad is to chromatically lower the seventh degree in major (second degree in minor) and to assign the root of a major triad to it: the resulting triad (in C major and A minor, Bb-D-F) is the only consonant triad whose root is a chromatically altered note and the other notes belong to the diatonic scale. The Neapolitan sixth is more frequently used in first inversion.

The **augmented sixth chords** involve an augmented sixth interval, and can be obtained from the chromatic raising of the second degree of the diatonic scale. In C major (A minor), the addition of D# produces the following chords:

- The **German sixth**, or **augmented sixth-five chord**: F-A-C-D#. It is the use of the dominant seventh chord as a vagrant chord (the seventh has been enharmonically changed).
  The connection between the German sixth to the Neapolitan sixth lying a fourth above (a fifth below) produces an authentic cadence in a different tonality (see figure 18.4). In this case, the German sixth behaves like a secondary dominant.
- The **French sixth**, or **augmented six-four-three chord**: F-A-B-D#. This chord is made up by two major thirds separated by a tone interval. It has two roots: F and B. In fact, the second inversion B-D#-F-A is another French sixth whose root lies a tritone upward or downward.
  The French sixth is strongly dissonant, and can be obtained e.g. from the dominant seventh chord by lowering the fifth, from the half-diminished seventh chord by raising the third, or from the minor seventh chord by raising the third and lowering the fifth.
- The **Italian sixth**: F-A-D#. It is a French sixth without one of the two roots.

In figure 18.3 examples of the vagrant usage of the major triad (Neapolitan sixth) and the dominant seventh chord (German sixth) are shown. Figure 18.4 displays some examples of the vagrant chords.

Further dissonant chords are produced by adding a minor seventh to the augmented triad, e.g. C-E-G#-Bb, or both a minor seventh and a minor or major ninth: C-E-G#-Bb-Db and C-E-G#-Bb-D. If the root is omitted in the last, the chord E-G#-Bb-Db is obtained: it comprehends a major third, a diminished fifth and a diminished seventh. Also these chords are usually adopted for obtaining vagrant harmonies.



[**Figure 18.3**: Examples of German sixth and Neapolitan sixth. The German sixth connects to the Neapolitan sixth through an authentic cadence, and a diminished seventh chord brings back to C major.]

The augmented triad, the French sixth, and the chords C-E-G#-Bb and C-E-G#-Bb-D discussed above, are examples of **whole-tone chord**s, i.e. the chords that can be obtained by taking some degrees from the **whole-tone scale**, a scale made up by tones only. Only two whole-tone scales exist:

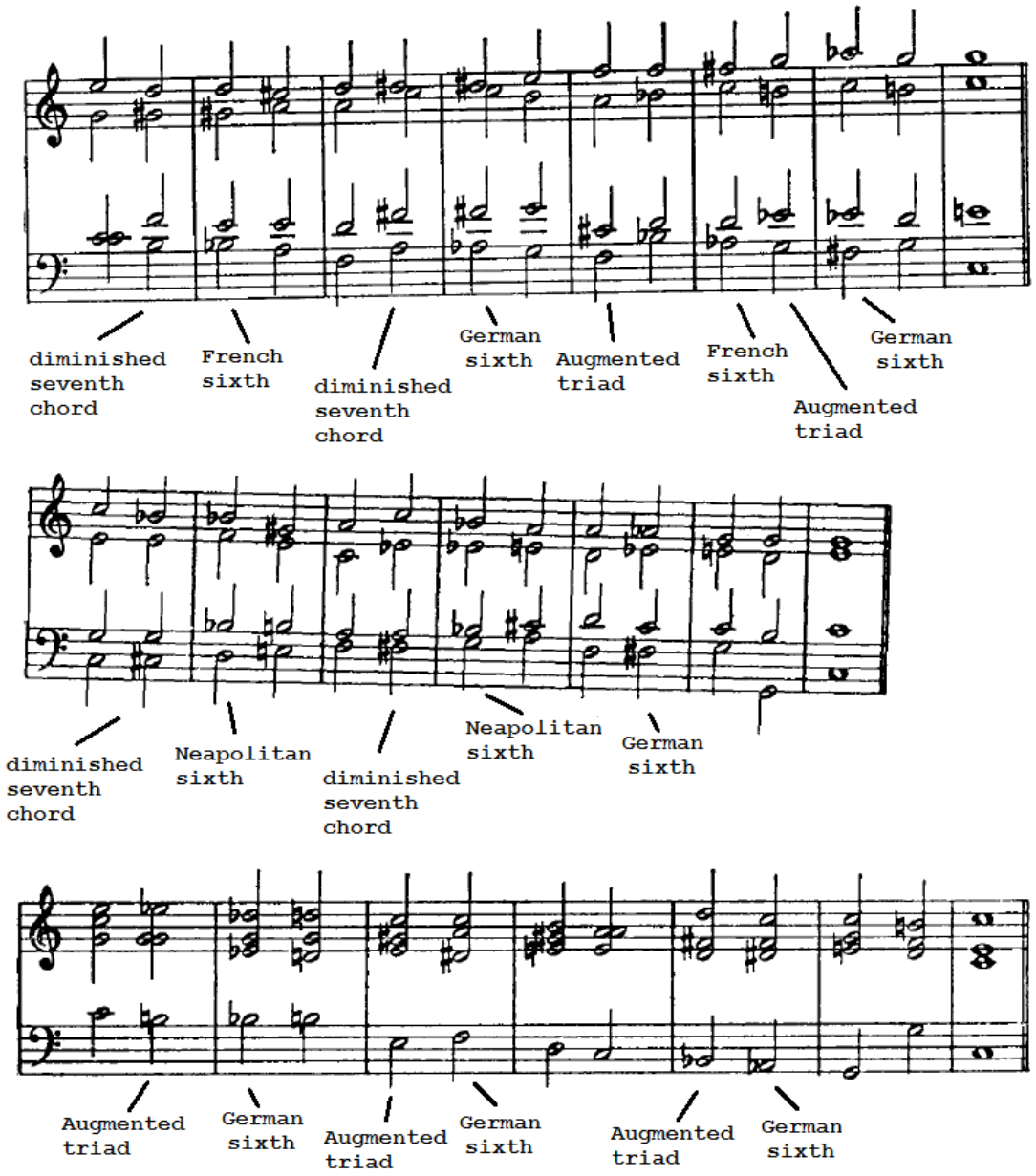
- C, D, E, F#, G#, A#, (C)
- Db, Eb, F, G, A, B, (Db)

No major, minor or diminished triads are possible when the whole-tone scale is adopted. This scale was frequently used by Debussy and other impressionist composers.

Another interesting scale is the **octatonic scale**, also called **diminished scale** or **whole-step-half-step** scale, that alternates tones and semitones (S-T-S-T-S-T-S-T). Three octatonic scales exist:

- C, C#, D#, E, F#, G, A, Bb, (C)
- C#, D, E, F, G, G#, A#, B, (C#)
- D, Eb, F, F#, G#, A, B, C, (D)

This scale contains all the intervals, and it is made up with the notes of two diminished seventh chords. It was frequently used by Rimsky-Korsakov and the other Russian masters of the XX century.



[**Figure 18.4**: Examples of vagrant chords. The scores have been taken from Arnold Schoenberg's *Theory of Harmony*.]

19-Chromatic modulations, modal mixture, suspended tonality

[This chapter is under construction]

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