Harmony

v_0.3

0-Introduction

Harmony is the manifestation of notes sounded simultaneously. The theory of harmony comprehends the fundamental elements of Western music, especially the common practice period (about 1650-1900).

Music can also exist in the form of a single melody and its rhythmic organization: many musical cultures, such as those of India and China, primarily oriented their developing to the horizontal dimension of the musical sound, the counterpoint. Counterpoint focuses on the melodies, which have an inner logic in terms of some scales (modes) out of which they are formed.

During the European Middle Ages and Renaissance, however, the research of conventions and handy rules for writing choral liturgical music led musicians to the idea that melody was only the uppermost layer of a piece of music. By means of uncountable attempts and experiments, the musical scales have been synthesized into a new and profound setup, the tonality. Tonality never stopped to be the central element of the most relevant Western music up to the 20th century.

Since the harmony involves the acoustic aspects of the chords which are produced through a superposition of different melodies, counterpoint and harmony cannot be treated as separated entities. It is also to be considered that, whenever some notes are sounded consecutively, the ear creates their simultaneity, since it perceives the harmony that would be produced if the notes had sounded together.

Although the acoustic properties of notes and chords cannot change, by analyzing the music of the tonal era it can be noted that many guidelines for the voice-leading, such as the treatment of dissonance, have been gradually unnoticed by the greatest composers. On the other hand, a significant amount of rules were introduced for meeting the needs of choral music, where the circumstances forced composers to obtain complexity only through the superposition of simple melodic lines. The historical development gives help in explaining how the natural material, i.e. the notes and their harmonics, have been brought into relation with the human perception of the sounds. Therefore, particular attention to the traditional four-part vocal writing will be given in this text, since it is the standard environment where the laws of the voice-leading best apply and have been historically introduced.

A primary role will be given to the appearance of the harmonics, which are involved in the sound produced by all traditional, analog or digital musical instruments. Their presence is a fundamental aspect of the human perception of the chords and the relationships between them. The analysis of the harmonics will provide some explanations to various characteristics of the tonal music, such as the minor tonality and the consonance or dissonance of the intervals.

Few remarks on the historical background where the theory of harmony has been developed are given in chapter 2. Furthermore, since harmony doesn't be the first subject to be examined when approaching the theory of music, a short overview of the music fundamentals and the common

notation is given in chapter 3. However, it is nothing more than an attempt to provide some completeness for the full text.

1-Harmonics and diatonic scale

Sounds are waves travelling through the air or some other medium. The musical notes, or *tones*, are sounds having one recognizable frequency. The frequency, also called the "pitch", is the highness of a sound. Technically, the frequency of a wave is the number of oscillations per second performed by the air molecules involved in its (mechanical) propagation, and is measured in Hertz (Hz). The oscillations' amplitude is related to the volume of the sound, and is measured in decibels (db).

Whereas the noises consist of a casual superposition of sound waves having different frequencies, a traditional musical instrument, except some percussions, is an object that produces a mixture of waves where a frequency which is much louder than the others can be detected: it is the frequency of the note that is sounded by the instrument.

An essential property of all musical instruments (as well as the vocal cords) is the following: in addition to the note's own frequency, also its multiples have considerable volumes. That is, if an instrument is playing e.g. the note A corresponding to the frequency 110 Hz, then also the frequencies:

110*2 =220 Hz, 110*3=330 Hz, 110*4=440 Hz, 110*5=550 Hz, …

are particularly emphasized. These frequencies are the **harmonics** or **overtones** of the note that is sounded. The list would be endless, but the volume of the harmonics diminishes (and the rate of such decrease is quite variable) as the frequency increases, so that only the initial harmonics can be detected. Usually the term "harmonics" comprehends the note itself, which is the 1st and loudest harmonic, while the term "overtones" refers to the rest of the harmonics. The 1st harmonic is called **fundamental frequency**.

Every instrument accentuates the multiples of the note that it's sounding, that is, instruments sounding the same note produce exactly the same list of harmonics.

The presence of the harmonics in the musical sounds is related to the mechanical properties of the instruments themselves, which are resonant objects. However, each instrument accentuates some harmonics more than others: that is, whereas the frequencies of the harmonics don't change if a different instrument is taken, their volumes are a peculiar aspect of each source of sound. The (infinitely populated) set of harmonics' volumes of an instrument is called **timbre**. The timbre makes a sound distinctive, so that it's possible to recognize the particular instrument that is sounding.

The differences between the timbres of different instruments can usually be neglected when writing music: the magnificence of a magnificent piece generally doesn't depend on the kind of instruments that are sounding. However, when putting the notes together for creating chords (a **chord** is a group of notes sounded simultaneously) and melodies, the presence of the harmonics

plays an essential role. The musical activities usually focus on the first 10-15 harmonics; in particular, the initial 6 multiples make the sound of the instruments particularly pleasant, and whenever higher harmonics prevail over them, the notes become harsh and dull.

Among the harmonics of a note, the frequencies which are multiplied by 2, 4, 8, 16, … (the powers of 2) correspond to very special harmonics: they are the **octave**s of the note. The sound produced by a note sounded along with one of its octaves is very distinctive: it's one of the most basic acoustic aspects of music.

Outside the score, the notes are indicated by the letters from A to G. The octaves of a note are associated to the same letter. That is, taking e.g. the note C as fundamental frequency, by multiplying its frequency by 2, 4, 8, 16, … another C is obtained. The lowest note that is used in music is the C that corresponds to about 16.35 Hz (hardly hearable by humans). In the American Standard (or Scientific) Pitch Notation, this C is denoted as C0. The octaves of C0 are indicated with C1, C2, C3, …, that is:

C1= C0*2, C2=C0*4, C3=C0*8, C4=C0*16, …

The octaves of C0 are used as a reference list for naming all the other notes: if the frequency of a note, say G, is comprehended between e.g. C3 and C4, this G will be called G3. This method allows to recognize each G among all its octaves. If the number is omitted, the notes are to be considered in the same octave (or in adjacent octaves).

All the harmonics of a note, and not only the octaves, can be associated to other notes. Taking e.g. C2 as fundamental frequency, the initial harmonics are:

C2, **C3**, G3, **C4**, E4, G4, Bb4, **C5**, D5, E5, F#5, G5, Ab5, Bb5, B5, **C6**, …

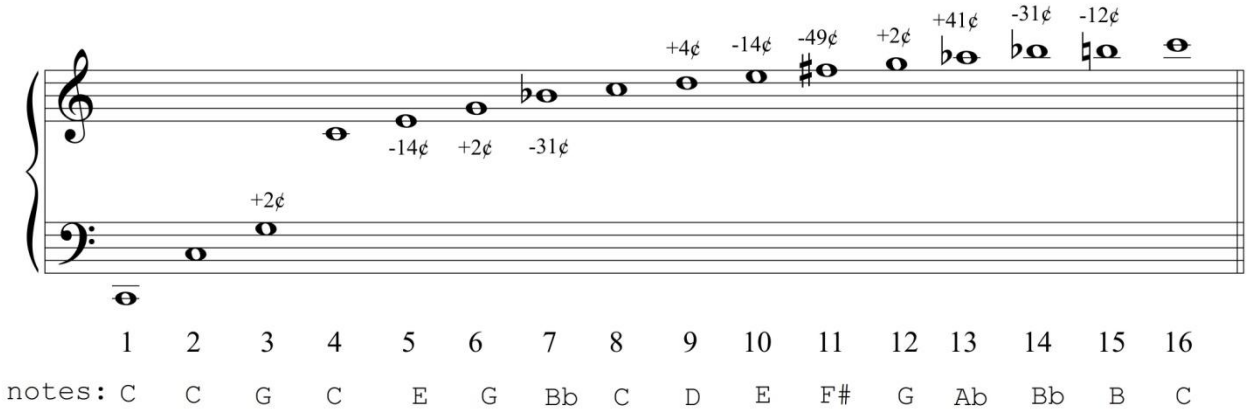
where the bolded letters mark the octaves of C2. This list correspond to the frequencies:

65.4, 130.8, 196.2, 261.6, 327.0, 392.4, 457.8, … (Hz)

However, if these frequencies are compared with the above notes sounded on e.g. a traditional piano, it will be possible to observe that, except the octaves, they don't be exactly the same pitch: the harmonics and the piano's notes differ for some amount (or lack) of Hz. For to assign each harmonic to a note, therefore, some approximation is required. A criterion for establishing these approximations is called **tempered system**. The **equally-tempered system** is by far the most used tempered system in Western (and also many non-Western) music.

The equally-tempered system divides each octave into 12 equal parts, called **semitone**s or "half-steps". Each half-step is itself subdivided into 100 cents (symbolized with ¢). Two consecutive half-steps, i.e. a "whole-step" (200 ¢), are called **tone**.

In the figure 1.1, the notes corresponding to the first harmonics are shown on the score, along with their equally-tempered system's approximations.



[**Figure 1.1**: Harmonics of C2 along with the corresponding equally-tempered system's approximations.]

The difference in pitch, or "distance", between two notes is called **interval**. The most important interval is between a note and its first octave (twice the note's frequency), which is itself called "octave". The octave interval exactly corresponds to the distance between the 1st and the 2nd harmonics, and also between the 2nd and the 4th, the 4th and the 8th, the 8th and the 16th, and so on.

Furthermore, the distance between the 2nd and the 3rd harmonics (e.g., in the above example, the interval C3-G3) is called "perfect fifth" interval or just "fifth", and the distance between the third and the fourth harmonic (G3-C4) is the "perfect fourth" interval or "fourth". These intervals, the octave, the fifth, and the fourth, occur in the music of almost all cultures. Other common intervals can also be found in the sequence of the harmonics: e.g., the "major third" interval (between the 4th and the 5th harmonics) and the "minor third" interval (between the 5th and the 6th).

As noted above, once a fundamental frequency is chosen, its harmonics are bound to it through a fixed and invariable mathematical relationship. Therefore, the intervals between the harmonics don't change if a different fundamental frequency is taken: an octave always lies between the 1st and the 2nd harmonics, a perfect fifth always lies between the 2nd and the 3rd, and so on. That is, every note brings with itself the same sequence of intervals.

Consequently, if the fundamental frequency is increased or lowered by exactly one or more octaves, then the notes associated to its harmonics remain the same: e.g., the harmonics of C2 are C2, C3, G3, C4, E4, G4, … and the harmonics of C3 are C3, C4, G4, C5, E5, G5, … In other words, the octaves of a note have the same harmonics of the note. This fact allows the brain to recognize the octave.

Whereas the intervals between the harmonics remain fixed while the fundamental frequency changes, the frequency-gap each interval corresponds to, on the contrary, doesn't be constant. E.g., C3 (130.8 Hz) is twice the frequency of C2 (65.4 Hz), and C4 (261.6) is twice the frequency of C3: so, the difference in frequency between C2 and C3 is half the difference in frequency between C3 and C4, but an octave interval continues to separate C1 from C2, and C2 from C3. In other words, each octave corresponds to a frequency-gap which is twice the range of the previous octave, and which

is half the range of the next. Consequently, also all the other intervals will change by a constant rate. E.g., each semitone interval increases in frequency by a factor of 1.059 Hz (the twelfth root of 2) with respect to the previous (adjacent) semitone, and the tones increase by a factor of 1.122 Hz (the sixth root of 2).

Thus, the interval between any two notes is related only to the *ratio* between the notes' frequencies. The human brain, in fact, recognizes the intervals between the notes without knowing their frequencies. When listening to a piece of music, the brain doesn't consider the pitch of each note: instead, it detects the ratios between the notes' frequencies, that is, the intervals. This evaluation is possible thanks to the presence of the harmonics: the brain compares the harmonics of two different notes, and from such comparison it gets the interval they form. If two different notes have the same harmonics, the brain recognizes the octave interval. Few people, however, have what it's called "perfect pitch" or "absolute pitch": when they listen to a frequency, they can recognize the note corresponding to that frequency.

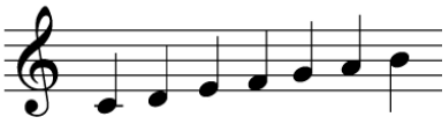
Before the researches of Helmholtz during the nineteenth century (that led to the publishing of his famous text *Sensations of Tone*), the presence of the harmonics hasn't been completely analyzed and comprehended. However, the harmonics have played a primary role since before the birth of harmony, when Western music was entirely based upon systems of scales.

A **scale** is a sequences of notes, arranged in ascending or descending order of pitch, that is used as a collection of sounds for making the melodies. In Western music, the scales generally consist of seven notes and their full length is one octave, so that they can be easily extended over different octaves. In particular, the **diatonic scale** is the most famous seven-notes scale. It derives from the old Greek theory: it could be obtained by the adding together two identical four-notes scales, called *tetrachords*, separated by a tone.

The diatonic scale that begins on the note C, also called "C diatonic scale", is:

C, D, E, F, G, A, B, (C)

and its notes correspond to the white keys of the piano. In the figure 1.2 these notes are displaced on the score.



[**Figure 1.2**: The diatonic scale.]

The intervals between E-F and between B-C correspond to a semitone, and the other notes lie a tone distant one from each other. That is, calling T the tone and S the semitone, the diatonic scale corresponds to the sequence of intervals:

T-T-S-T-T-T-S

where the sum of tones and semitones is the amplitude of the octave. Each note of the scales is also called **degree**. In the case of the above C diatonic scale, C is the first degree, D is the second, E is the third, and so on.

Although the precise derivation of the diatonic scale from acoustic aspects is matter of discussion (see chapter 5), this scale is strongly related to the perception of the harmonics in the musical sounds: it comprehends all the first 10 harmonics of C but Bb.

By switching the positions of the 5 tones and the 2 semitones of the diatonic scale, many different scales can be produced in the range of one octave. In particular, in the European music before the sixteenth-seventeenth centuries, the scales (that were called *modes*) were produced by maintaining unaltered the order of the C diatonic scale's tones and semitones, and beginning a new scale from each degree. The result is:

Ionian:	C, D, E, F, G, A, B, (C)	(T-T-S-T-T-T-S)
Dorian:	D, E, F, G, A, B, C, (D)	(T-S-T-T-T-S-T)
Phrygian:	E, F, G, A, B, C, D, (E)	(S-T-T-T-S-T-T)
Lydian:	F, G, A, B, C, D, E, (F)	(T-T-T-S-T-T-S)
Mixolydian:	G, A, B, C, D, E, F, (G)	(T-T-S-T-T-S-T)
Aeolian:	A, B, C, D, E, F, G, (A)	(T-S-T-T-S-T-T)
Hipophrygian:	B, C, D, E, F, G, A, (B)	(T-S-T-T-S-T-T)

where the first one (the Ionian mode) coincides with the diatonic scale itself. Although the classification of the modes varies according to the historical period, the above seven scales are known as **diatonic modes**. The interaction between tones and semitones provides a particular mood or feeling to the melodies that are based on each mode.

In all the diatonic modes but the last one (which begins on B), the distance between the first and the fifth degrees is a fifth interval. The acoustic affinity between a couple of notes separated by a perfect fifth is greater than that of any other interval, except the octave. In fact, consider e.g. the C-G fifth interval. The initial harmonics of C are:

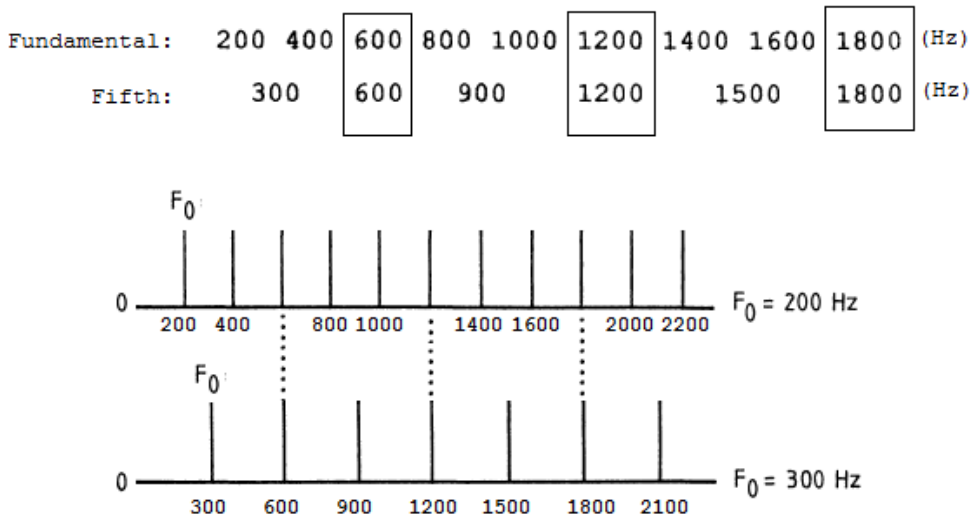
C2, C3, **G3**, C4, E4, **G4**, Bb4, C5, **D5**, E5, F#5, **G5**, Ab5, …

and those of G are:

G2, **G3**, D4, **G4**, B4, **D5**, F5, **G5**, A5, …

Look at the bolded notes: the 2nd harmonic of G coincides with the 3rd harmonic of C, the 4th harmonic of G coincides with the 6th of C, the 6th harmonic of G coincides with the 9th of C, and so on (see also the figure 1.3). Thanks to this (exceptionally) large amount of harmonics that C and G

have in common, the brain easily recognizes the fifth intervals when these notes (and every other couple of notes a fifth apart) are listened.



[**Figure 1.3**: Harmonics that two notes a fifth apart have in coomon.]

2-Brief history of harmony

The medieval scales, also called *Gregorian modes* or *church modes*, have been introduced in the European music by Pope Gregory I (r. 590-604). These scales allowed to collect and organize the body of the liturgical chants in use at the time, and remained the essential theoretical construct until the sixteenth century.

The modes are derived from the old Greek theory, where the seven notes of the diatonic scale could be obtained from the combination of two identical 4-notes scales (tetrachords). In fact, the diatonic scale can be produced by adding together two tetrachords whose sequence of tones (T) and semitones (S) is T-T-S, separated by a tone placed in the middle: T-T-S-(T)-T-T-S. As discussed in the previous chapter, the modes differed one to each other because they started on different diatonic scale's notes, so that the two semitone intervals were placed in different positions.

In the first centuries after the introduction of the modes, the Western musical practice have mostly neglected the possibility of superposing melodies. Among the first available examples of polyphonic music, the book *Enchiridion Musicæ*, written by the Flemish monk Hucbald (840?-930-32?), describes two melody proceeding in parallel and separated by a fourth, a fifth or an octave interval. This technique was called *parallel organum*. A century later, the Italian monk and theorist Guido d'Arezzo (990-1050) synthesizes many elements of various notational practices in his *Micrologus* and *Antiphonarium*. He introduces the modern staff notation, that replaced the neumatic notation. Behind the parallel organum, he includes oblique (diverging or converging) motion and contrary motion of voices.

Until the end of the 11th century, the note-against-note technique (i.e., two synchronized melodies that proceed with repetitive rhythm) is the only practice to be adopted. This composing style, where simultaneous notes create a sequence of equally time-spaced intervals, is called *homophony*. With the adventure of 12th century, it arises the usage of superposing a faster melody against a slower bass.

In middle of the thirteen century, the theorist Franco of Cologne establishes the first classification of the consonant intervals, distinguishing between "perfect consonances" (the octave only), "middle consonances" (fourths and fifths), and "imperfect consonances" (major and minor thirds). He consider the sixths dissonant intervals, but declares that their dissonance is less strong than that of augmented fourths, augmented fifths, and sevenths. Marchetto of Padua, who also lived in the 13th century, specifies that the dissonant intervals need special treatment: these intervals should be followed by consonant ones, a practice that in the following centuries will be known as *resolution* of the dissonance. Moreover, he classes the fourth among the dissonant intervals (this is not wrong, as it will be noted in chapter 4).

Among the most relevant contributions of the fourteenth century, in France and Italy it is remarkable the work of Philippe de Vitry, Guillaume de Machaut, Jacopo da Bologna, Francesco Landini, Jean de Muris, Ghirardello da Firenze and others. They are exponents of the *Ars Nova*

("new art") musical style, characterized by the increased use of smaller note values and new rhythmic solutions. The designation "Ars Nova", to be specific, was introduced around 1320 by de Vitry as the title of its four-manuscripts treatise. This development was in part anticipated by Pierre de la Croix in the second half of the thirteen century.

By the fifteenth century, under the influence of Britannic composers such as John Dunstable, the third and the sixth intervals become more largely adopted in the European music. In this period, Johannes Tinctoris (1436-1511) writes one of the earliest dictionaries of musical terms, entitled *Terminorum musicae diffinitorium*.

The evolution of harmony from the system of church modes encompasses several centuries and various styles of music. Until the sixteenth century, music was regarded as it were horizontally, and the chords were only an accidental result of the cohesion of independent voices. Behind the gradual growth of the perception of harmonic relations and the vertical dimension of simultaneously sounded melodies, the ecclesiastical scales have been progressively modified by the introduction of accidentals. This led to the synthesis that fused the various modes into the current major and minor scales, the basic elements of tonal music.

Whereas the ecclesiastical environment continued to sustain the production of music, during the Renaissance (about 1450-1600) the musical activities gradually shifted to the courts, where the finest composers wrote both secular pieces and sacred works for the court chapel. Both in the Middle Ages and the Renaissance, vocal music was more important than instrumental music, and choirs usually sung *a cappella*, i.e. without instrumental accompaniment. During the sixteenth century, however, instrumental music became less subordinate to the vocal practice, and more works specifically for instruments began to appear.

Many of the most relevant Renaissance musicians came from Italy, the Flanders (which today includes parts of the Netherlands, Belgium, and France), Germany, England, and Spain. Among them, there were Josquin Desprez (1440-1521), Thomas Tallis (1510-1585), Giovanni Pierluigi da Palestrina (1525-1594), Roland de Lassus (1532-1594), Giaches de Wert (1535-1596), William Byrd (1543-1623), Tomás Luis de Victoria (1548-1611), Luzzasco Luzzaschi (1545-1607), Luca Marenzio (1553-1599), Giovanni Gabrieli (1557-1612), John Dowland (1563-1626), Carlo Gesualdo (1566-1613), Thomas Campion 1567-1620), and Claudio Monteverdi (1567-1643), whose innovative work is largely held responsible for the change from the Renaissance style to that of the Baroque period.

Since the beginning of Renaissance, various theorists also have contributed. Important treatises are those of Gioseffo Zarlino (author of *Istitutione harmoniche*, 1558), Johann Lippius (*Synopsis musicae novae*, 1612), Adriano Banchieri (*Cartella musicale*, 1614), Michael Praetorius (*Syntagma Musicum*, 1619), Lorenzo Penna (*Li primi albori musicali*, 1672), Giovanni Bononcini (*Musico Prattico*, 1673), Angelo Berardi (*Il perchè musicale*, 1693), Francesco Gasparini (*L'armonico pratico al cimbalo*, 1708), Antonio Bruschi (*Regole per il contrapunto e per l'accompagnatura del basso continuo*, 1711), and François Campion (*Traité d'accompagnement et de composition, selon la regle des octaves*, 1716).

Finally, the monumental *Traitè de L'armonie* (1722) by Jean-Philippe Rameau is one of the most important treatises in the history of music. Although after Rameau many improvements have been made, his work comprises the foundations of modern harmony.

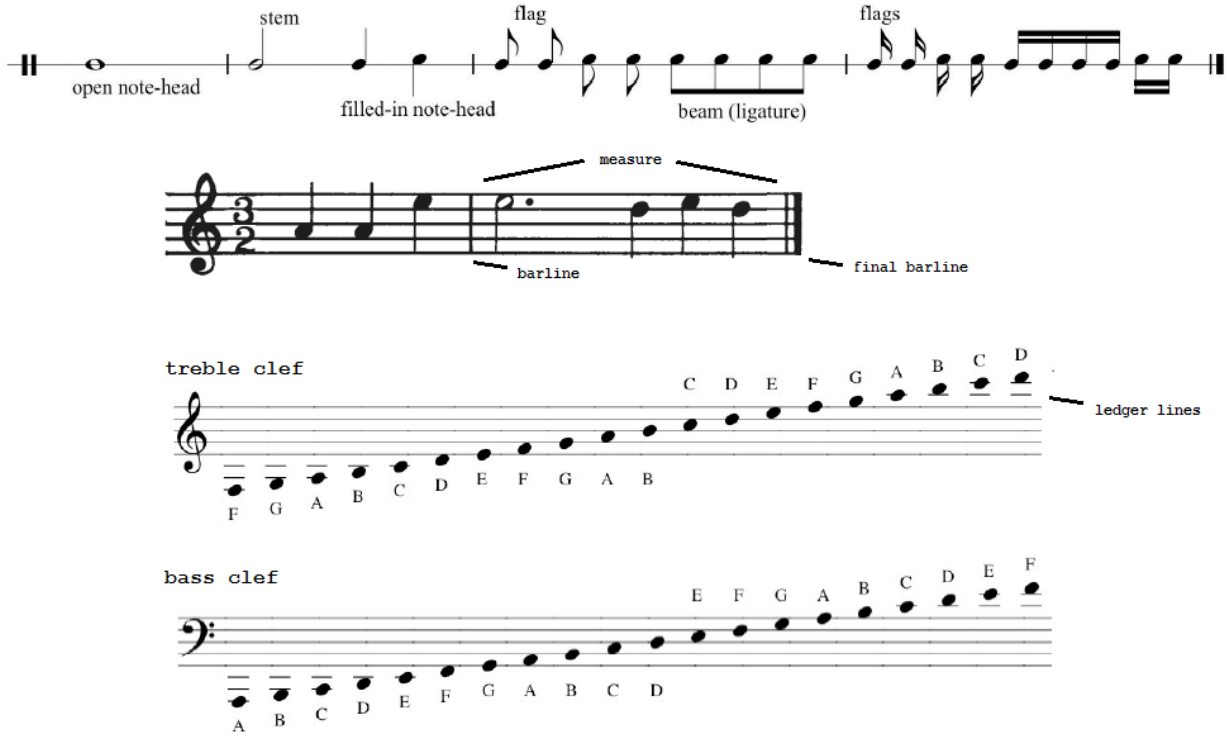
With the establishment of the principles of harmony in the theoretical background of polyphonic music, a new and deeper awareness of the sound phenomena led the composers of the common practice period to the production of artistic works of great complexity and perfection. Too many wonderful examples can be found among the opuses of the masters - from Bach to Debussy. There is no need to list them.

3-Music fundamentals and notation

Although many types of music notation have been introduced through the history of written music, the so called **common notation** is currently the most widespread format for writing music. Since the notation is also related to the technical aspects characterizing each musical instrument (or voice), the symbols and formats listed below are subjected to a great variety of additions that cannot be described in only one chapter.

Notes are associated to both a pitch and a time duration. They are written on the **staff** (plural=*staves*), where the time flowing is represented on the horizontal dimension (going from the left to the right), and the pitch is related to the vertical dimension. The 5 horizontal parallel lines composing the staff are the reference heights for the pitches: higher-pitched notes are located higher on the staff. If many staves have to be played at the same time, they are connected by a long vertical line at the left hand side.

Notes are indicated by drawing little ovals, called **notehead**s, which can be empty or filled-in. A **stem** is eventually attached to them, and one or more **flag**s on the top of the stem can be present. The noteheads can be placed both on the lines and in between two lines; for extending the staff higher and lower, small lines called **ledger lines** are to be used. See figure 3.1.



[**Figure 3.1**: Notes' symbols and names, ledger lines, measures, final barline.]

A **melody** (also called *melodic line*, musical phrase, or *theme*) is a sequence of notes that has some aesthetic purpose. The rhythm of a melody is the placement of the notes in time, which is obtained by assigning a **rhythmic value** or **note value** to the notes: the rhythmic value indicates the notes' relative durations with respect to the other notes. The actual time-extent of the notes depends on the speed at which the song is played. The presence of empty or filled-in noteheads, flags and stems allows to identify the different rhythmic values that the notes can assume.

The biggest rhythmic value is called **double whole-note** or **breve**. The other rhythmic values are submultiples of the breve: to be specific, they correspond to one breve divided by the powers of two, that is, a **whole-note** is half the length of a breve, a **half-note** is half the length of a whole-note, a **quarter-note** is half the length of a half-note (i.e., whole-note/4), an **eight-note** is half the length of a quarter-note (whole-note/8), a **sixteenth-note** is half the length of a eight-note (whole-note/16), and so on.

Notes will be drawn with the stem if their rhythmic value is smaller than the whole-note, and they are filled-in ovals if their rhythmic value is smaller than the half-note; whenever the notes are smaller than the quarter-note, the stem can have one or more flags (one flag the eight-note, two flags the sixteenth-note, and so on). See the figure 3.2.

The stems (along with their flags) can be equally drawn either up or down: in the first case the stem is attached to the right side of the oval, in the second case to the left. One can often see the note's stems being connected together by (roughly horizontal) lines called **beam**s - joining the note's flags perpendicularly to the stems. Beams are only used to make the rhythmic subdivision easier to read.

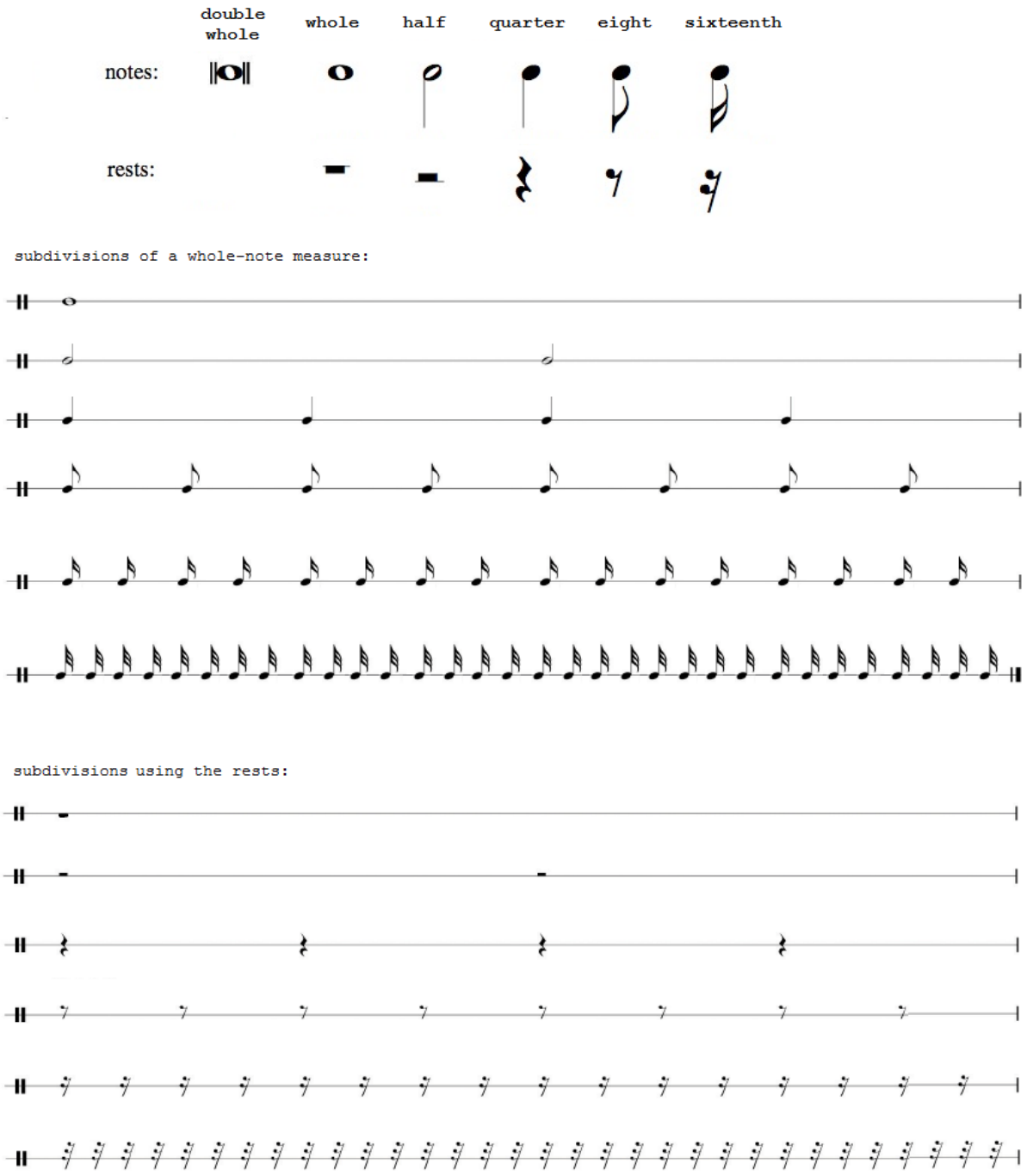
A pause (i.e., the silence) is called a **rest**. Rests' rhythmic values are catalogued in the same way as the notes (as shown in figure 3.2).

For to assign the diatonic scale's notes C, D, E, F, G, A, B to the staff lines, the **treble clef** and the **bass clef** symbols are the most commonly used. They are written at the beginning of each staff's row; starting from the bottom line of the staff, which is identified as the 1st line, the treble clef is placed on the second line, which becomes G: then, e.g. the third line will become B and the space in between the second and the third lines is A (see figure 3.1). The bass clef is written on the fourth line, which becomes F (and the same criterion applies to the other notes). Although these clefs can be placed on different lines, it is rare to find them anywhere but in these positions.

The staff is divided into short sections called **measure**s or **bar**s separated by vertical lines called **barline**s. On the composition's ending the **final barline** symbol is written. The sum of the rhythmic values of the notes and the rests laying in any measure must correspond to the measure's rhythmic value.

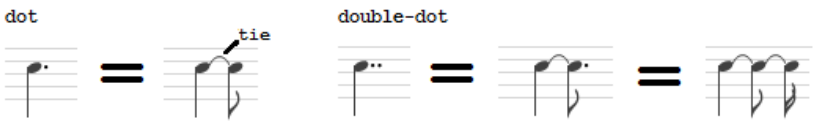
For joining the notes together to form a continuous sound a curved line called **tie** is used (see figure 3.3). On ore more notes of any length may be tied together; the rhythmic value the tied

notes stand for will be the rhythmic value of all the notes added together. Tied notes can be used e.g. for sustaining the note while going from one measure to the following.



[**Figure 3.2**: Rhythmic values of notes and rests, subdivisions of a whole-note measure.]

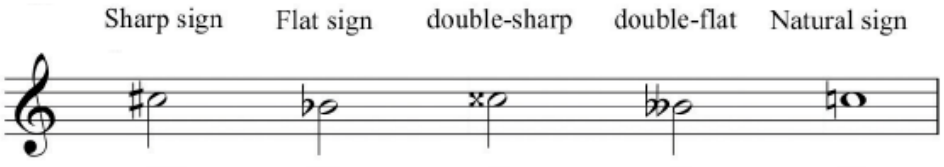
If some note has to assume one time and a half its rhythmic value, a **dot** right after the notehead can be added. For incrementing the rhythmic value by 3/4 of its value, the **double-dot** is used: the double-dot symbol, therefore, is equivalent to putting a dot to the increment provided by the first dot. Observe that a dotted note is the same as a note and a half-valued note tied together.



[**Figure 3.3**: Dot, double-dot, tie.]

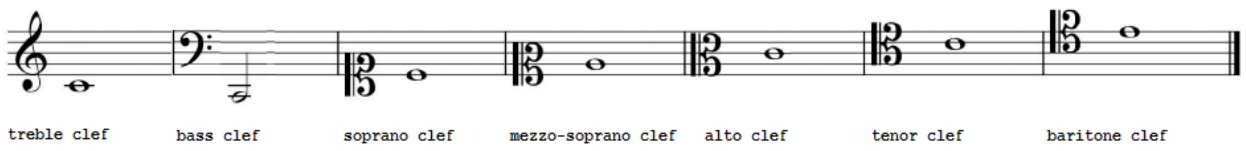
The **sharp** (#) and the **flat** (b) symbols, which are placed right before the noteheads, indicate that the note is to be considered respectively a semitone higher or a semitone lower. There also exist symbols for the whole tone-shifting: the **double-sharp** (similar to an "X") transports the note a tone higher, and the **double-flat** (bb) a tone lower. These four semitone-shifts and tone-shifts are called **accidentals** (shown in figure 3.4). If an accidental is placed upon a certain note, then, inside the measure where the note is found, all the following manifestations of such note are affected by the accidental. This happens unless another special symbol is placed: the **natural symbol**, which cancels the accidentals being written on its own line (or space) and measure before it. There is no need to pitch-back the raised/lowered notes when passing from a measure to the following one because the barlines don't keep alive both the accidentals and the natural signs.

Some special accidentals are placed at the beginning of each staff's row, instead of being found inside the measures. They are positioned right after the clef symbol: they are "active" through all the row, along with all their octaves, and the natural sign cancels them only in the measure and line/space where it is placed. The list of these accidentals is called **key signature**. The key signature can change during the piece; as we'll see in chapter 6, it is related to the choice of the tonality, and it is composed either by flats only or by sharps only (their quantity depends on the tonality).



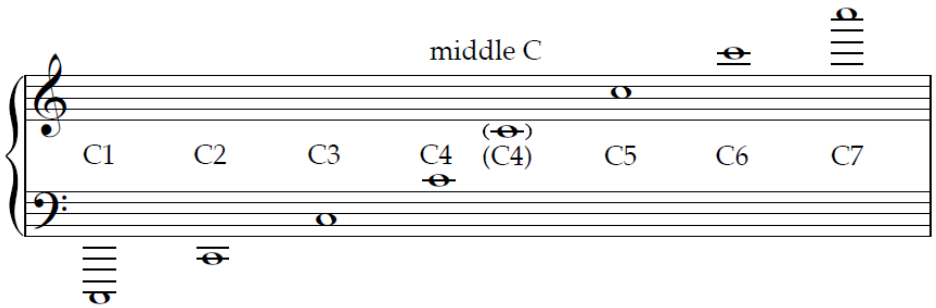
[**Figure 3.4**: Treble clef with accidentals and the natural symbol.]

The set of pitches, from lowest to highest, that an instrument can sound is called **range** of the instrument, and a specific part of the range is called **register** (e.g., the range can be divided in bass register and high register). Since instruments and voices have different ranges and registers, another clef symbol exists, the **C clef**, which is moveable and makes its staff line to be C. The C clef is called differently depending on the line upon which is placed, and its names coincide with the vocal registers: **baritone clef** (fifth line), **tenor clef** (fourth line), **alto clef** (third line), **mezzo soprano clef** (second line) and **soprano clef** (first line). Instruments with high ranges usually adopt the treble clef, instruments with low ranges usually adopt the bass clef, and transposing instruments (for which the notes are written higher or lower than where they sound) usually adopt the C clef. The list of the clefs is given in figure 3.5.



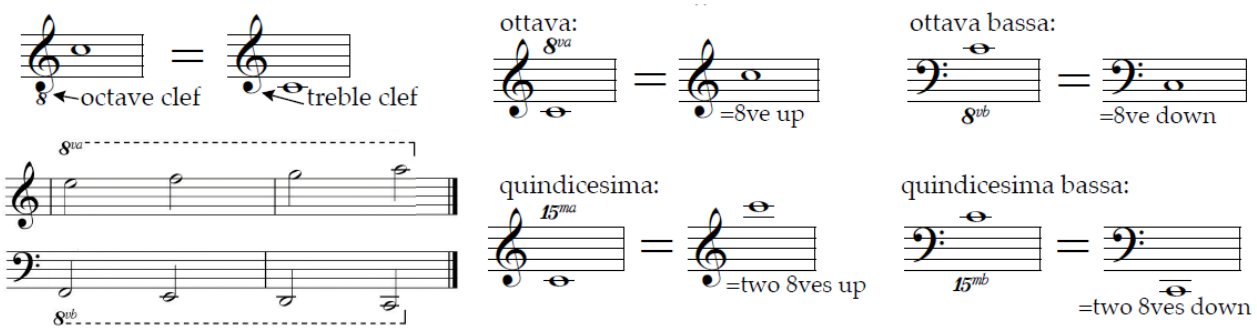
[**Figure 3.5**: Treble clef, bass clef, C clefs.]

In the case e.g. of scores for piano or for choir, the **grand staff** is used: it is composed by both the treble clef and the bass clef, and the notes can span through the two staves. The middle C4 is notated differently depending on whether it belongs to the treble or bass clef (see figure 3.6).



[**Figure 3.6**: Grand staff with middle C4.]

Among the pitch-related symbols, there exist various styles to indicate whenever notes are to be considered one (or more) octave above or below (see figure 3.7):

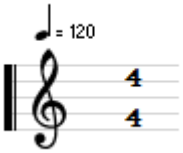


[**Figure 3.7**: Ottava, Octave clef, Quindicesima.]

In almost all Western music, time is organized by regular periods or **beat**s, so that the song can be seen as a flowing of equally time-separated pulses. E.g., dancers usually emphasize the rhythm through repetitive body-movements synchronized with the beats. The notes' rhythmic values are related to the rhythmic value of the beats, which is called the **beat unit**: if the quarter-note receives e.g. one beat, the half-note will receive two beats, and the whole-note four beats.

The **tempo** is the speed of the beats, i.e. their time duration, given by the number of **BPM** (Beats Per Minute) at the beginning of the score (and, eventually, at any spot where it changes). See figure 3.8. E.g., if one writes that a quarter-note (the beat unit) is equal to 60 BPM, then there are 60

quarter-notes in one minute, so that each quarter-note corresponds to 1 second. Thus, the greater is the number of BPM the faster is the song.



[**Figure 3.8**: Example of tempo marking.]

Since the performers often give emphasis to the music by slightly changing the tempo, many Italian terms are also written above the notes. From slowest to fastest, some of them are the following: *Grave* (solemn), *Largo* (very slow), *Lento* (very slow), *Adagio* (slow), *Andante* (moderately slow), *Moderato* (moderate), *Allegro* (fast), *Vivace* or *Vivo* (lively), *Presto* (very fast), and *Prestissimo* (very fast). There also exist terms for marking a gradual changes of the speed, such as *accelerando* (abbreviated *accel.*, accelerating, getting faster), *ritardando* (abbrev. *rit.*, slowing down) and *rallentando* (abbrev. *rall.*, gradually slower).

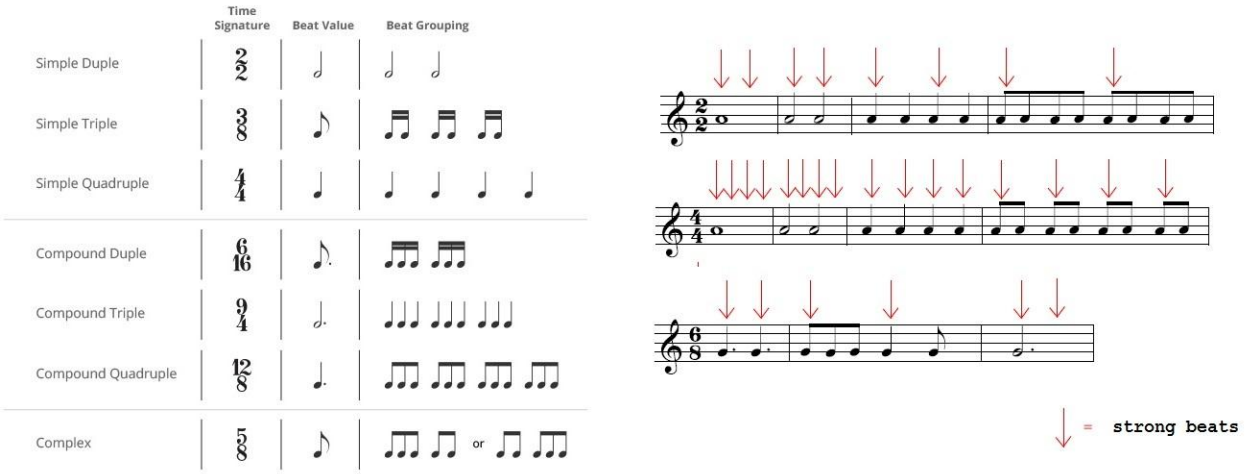
The beats lying in each measure are virtually divided into groups, usually including two, three or four beats, so that a repetitive pattern arises when many consecutive measures have the same subdivision. The most important beat of any measure is the first one, which is called the **downbeat**. The downbeat and the first beat of any group are called **strong beat**s, while the others are the **weak beat**s. The notes synchronized with the downbeat and the other strong beats are to be emphasized by the executor to make explicit the subdivision. The **time signature** or **meter signature** indicates how the beats are grouped, so that it specifies the way the strong and the weak beats are displaced within the measures.

The time signature is represented by a "fraction", where the upper value is the number of beats which are present in the measure, and the bottom number is the beats' own rhythmic value (the beat unit). The upper number also establishes the way the beats are divided into groups:

- If the upper number is a multiple of 2 and not a multiple of 4 (i.e., usually 2 and 6) the meter signature is a **duple meter**, and the measure's beats are divided into 2 groups.
- If the upper number is a multiple of 4 (usually 4, 8 or 12) the meter signature is a **quadruple meter**, and the measure's beats are divided into 4 groups.
- If the upper number is a multiple of 3 and not a multiple of 2 or 4 (usually 3 and 9), the meter signature is a **triple meter**, and the measure's beats are divided into 3 groups.
- All the other cases are called **asymmetric time signatures**: the measure includes a combination of different groups.

This subdivision is related to the number of groups that the beats are divided into: duple meters have 2 groups, triple meters have 3 groups, and quadruple meters have 4 groups. The first beat of each group is a strong beat, the others are weak beats. Some examples are shown in figure 3.9.

E.g., the 3/2 time signature (a triple meter) corresponds to 3 groups of 1 half-note beat, the 8/4 time signature (quadruple meter) corresponds to 4 groups of 2 quarter-note beats, the 6/8 time signature (duple meter) corresponds to 2 groups of 3 eight-notes, and the 3/4 time signature (triple meter, the waltzes' rhythm) involves 3 groups of 1 quarter-note beat. Common examples of asymmetric time signatures are 5/8, which is decomposed into a group of 3 beats and a group of 2 beats, and 7/8, whose group-subdivision can be 2+2+3, 3+4, 2+3+2, ….



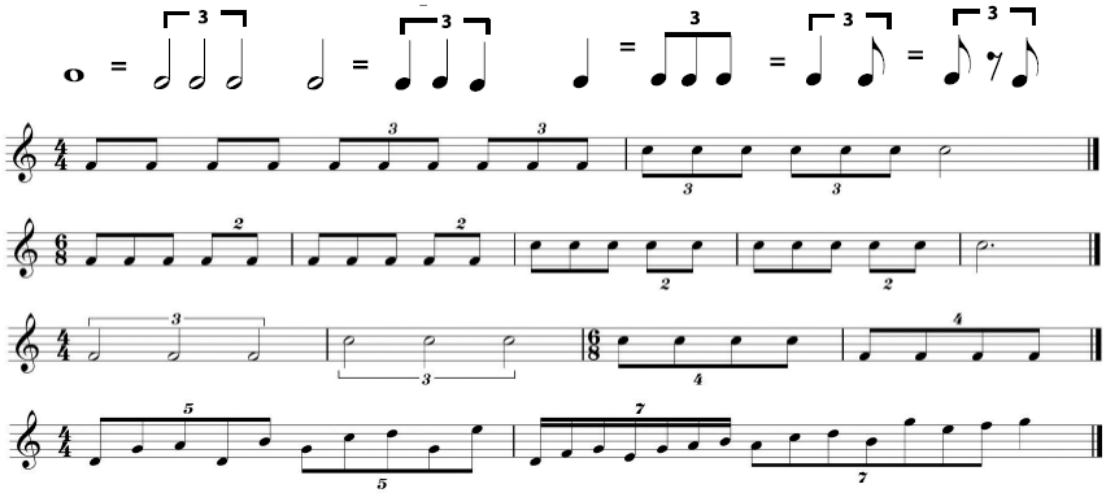
[**Figure 3.9**: Examples of time signatures, beats grouping in different meters, strong beats.]

The 2/2 and 4/4 time signatures, respectively called **cut time** and **common time**, are of very common usage. They are often written by drawing respectively the symbols of a vertically barred "C" and a "C" in place of the respective fractions.

Thus, the meter signature determines how the pulses are grouped together in discrete segments, and the tempo establishes their time duration. Different combinations of tempo and meter signature can provide the same setting for the composer: e.g., assigning a quarter note to 60 BPM and adopting the 3/4 time signature is the same as assigning a quarter note to 120 BPM and adopting the 3/2 time signature. Normally, the time signature that makes the music easier to read is adopted. The meter can change during the song; in such cases, the new meter signature will be written at the beginning of the first measure after the change (see figure 3.10). There also exist compositions, such as the ancient Gregorian chants and some non-Western music, where no meter can be identified.

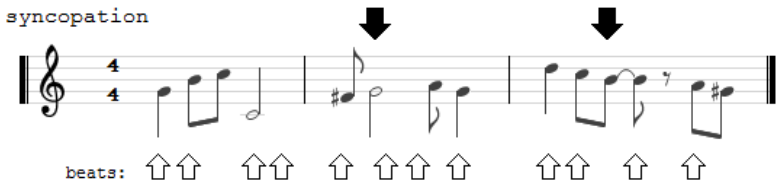
It often arises the necessity of dividing a note (its rhythmic value) into anything other than 2, 4, 8, … (the powers of two), as well as placing three equal notes inside the duration of 2, 4, 8, … beats. These **borrowed division**s can be obtained by using the **tuplet**s or **grouplet**s symbols. Among them, the **triplet**s are the most used. Triplets divide the rhythmic values (of a beat or a group of 2, 4, 8, … equal notes) into three equal portions (rather than into powers of two). In practice, one consider the rhythmic value that has to be divided as it was dotted, so that it contains three halves of itself, and places a bracket above it (see figure 3.10). The **duplets** and **quadruplets** make the opposite operation of creating respectively 2 and 4 equal rhythmic values into a group of three

beats. There also exists **quintuplets**, **sextuplets** and **septuplets** too, where the same criterion applies.



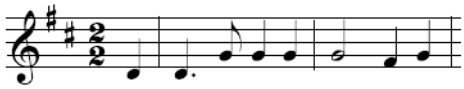
[**Figure 3.10**: Examples of borrowed divisions.]

The **syncopation** or **syncopated rhythm**, one of the most customary features of Western music, occurs when an arrangement of notes places accent on a weak beat (or weak element of the beat's subdivision), instead of being synchronized with the strong beat, so that the rhythm puts an emphasis when it is unexpected. E.g., a "long" note that is placed on a weak beat creates a syncopation. See e.g. the figure 3.11.



[**Figure 3.11**: Examples of syncopation.]

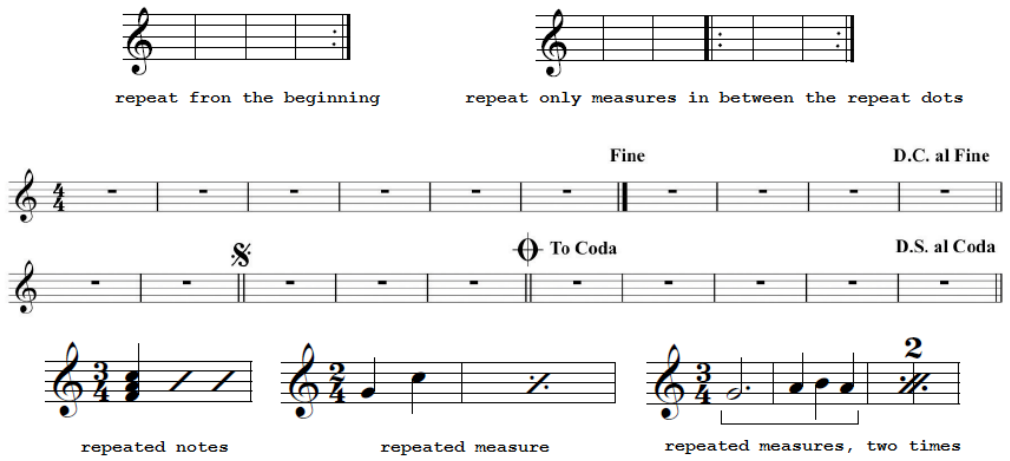
Sometimes happens that the composition begins before the first barline, in the middle or at the end of a shorter measure placed right before the first measure. This usage is called **anacrusis** or **pickup measure** (see figure 3.12).



[**Figure 3.12**: Anacrusis.]

Whenever a measure or a group of subsequent measures has to repeated, the **repeat dots** are placed at the beginning and at the end of the involved section, respectively behind the first and the

last barlines (if no beginning repeat dots are present, the performer has to go all the way back to the beginning of the score and repeat from there). There are some tricks to denote the repetition of sections, e.g. to put a couple of particular reference symbols (see figure 3.13) and to write *D.C.* and *al fine*, or *D.S.* and *al coda*: "D.C." means repeat back to the beginning; "al fine" is often used in conjunction with D.C. and appears at the point which tells the performer to end the piece; "D.S." means repeat back to the symbol; "al coda" is often used in conjunction with D.S., and tells the performer to "jump" to a separate final section. Moreover, it is possible to explicitly write a number of measures that corresponds to the duration of the repeated section, but without filling them with the notes: in place of the notes, special symbols are placed, as shown in the figure.

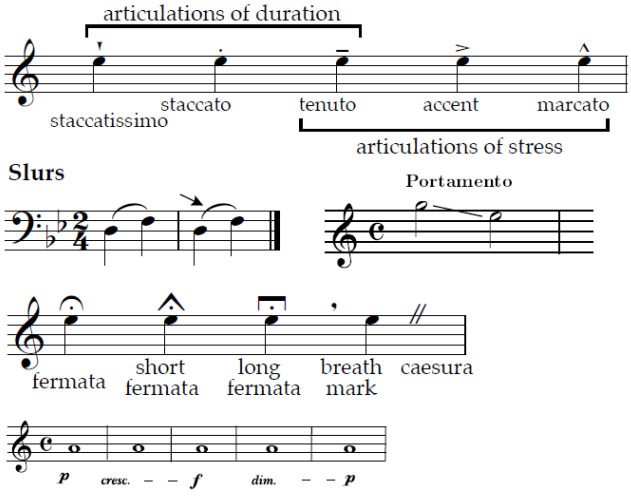


[**Figure 3.13**: Commonly used repeat signs.]

A quantity of signs that indicate how to play one or more notes also exists. The **dynamics** marks indicate the relative loudness of the notes, and writing *crescendo* vs. *diminuendo* (abbreviated *cresc.* and *dim.*) indicates respectively to increase or to decrease the loudness while playing group of notes (see figure 3.14).

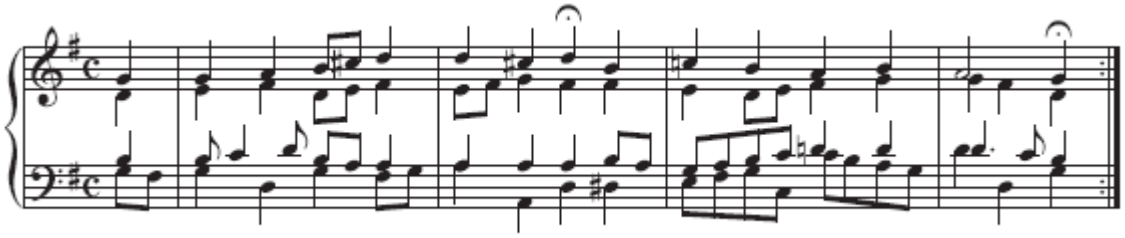
Dynamics

ppp = very, very soft
pp = **pianissimo** = very soft
p = **piano** = soft
mp = **mezzo-piano** = medium-soft
mf = **mezzo-forte** = medium-loud
f = **forte** = loud
ff = **fortissimo** = very loud
fff = very, very loud
fp = forte followed suddenly by piano; also ***mfp***, ***ffp***, etc.
sfz = sforzando = a forceful, sudden accent
fz is forceful but not as sudden as ***sfz***



[**Figure 3.14**: Dynamics, articulations, slurs, portamento, fermata.]

The **articulations** are used to specify how the duration or the stress of a group of notes should be performed; they strongly depend on the particular instrument that is making the music. Among the most commonly used, *staccato* indicates that the notes are to be sounded shorter than their own rhythmic value (and *staccatissimo* means "more staccato"); *legato* or *tenuto* is the opposite of staccato (the notes are very connected). An **accent** requires that the notes stand out more than the other notes around them, and *marcato* means "marked": the notes are enough accentuated and/or spaced to be better perceived. The **slurs** are signs for connecting different pitches (in some way, depending on the instrument that is making the music), and the **portamento** is a slide between the two notes including all the pitches in between. For concluding, the **fermata**s signs indicate to keep the note until the conductor or soloist do something.



[**Figure 3.15**: Beginning of a Bach's chorale.]

4-Intervals and scales

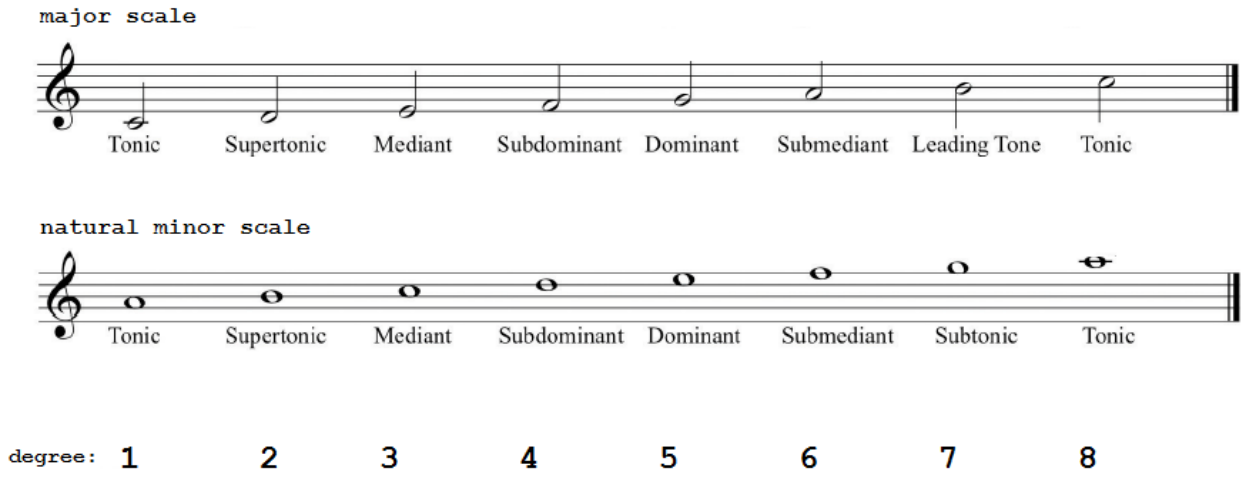
When beginning on C, the diatonic scale has the form:

C, D, E, F, G, A, B, (C)

The diatonic scale coincides with the **major scale**. The same seven notes are involved in the **natural minor scale**:

A, B, C, D, E, F, G, (A)

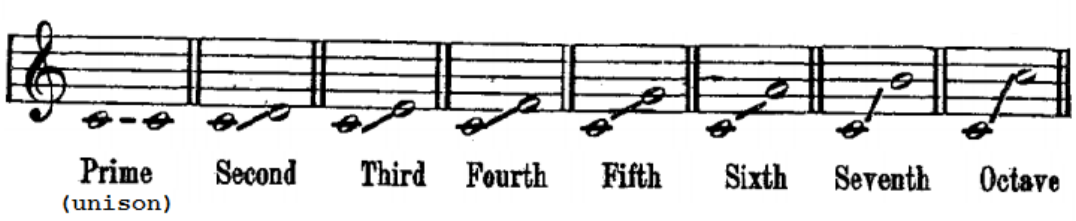
The beginning notes of these scales, respectively C and A, are called **tonic** or **fundamental tone**. The second degree is called **supertonic**, the third **mediant**, the fourth **subdominant**, the fifth **dominant** and the sixth **submediant**. The seventh degree of the natural minor scale is called **subtonic**, and the seventh degree of the major scale is the **leading-tone**.



[**Figure 4.1**: C major scale and A natural minor scale.]

The mediant, the submediant and the subtonic degrees of the natural minor scale lie one semitone lower than the mediant, the submediant and the leading-tone of the major scale. The other degrees are coincident, i.e. they form the same intervals with their tonic. In particular, the amplitude of the tonic-mediant interval distinguishes a major scale from a minor scale: whenever it corresponds to 4 semitones, as in the major scale, the scale is a major or "major-like"; if it corresponds to 3 semitones, as in the natural minor scale, the scale is a minor or "minor-like".

The intervals between the diatonic scale's notes are called *prime*, *second*, *third*, *fourth*, *fifth*, *sixth*, and *seventh* depending on the number of degrees they occupy, including both the notes forming the interval. E.g., the fourth interval D-G occupies 4 degrees: D, E, F, G. The prime interval is the interval between a diatonic degree and itself. The intervals between the tonic and the other notes of the diatonic/major scale are shown in figure 4.2.



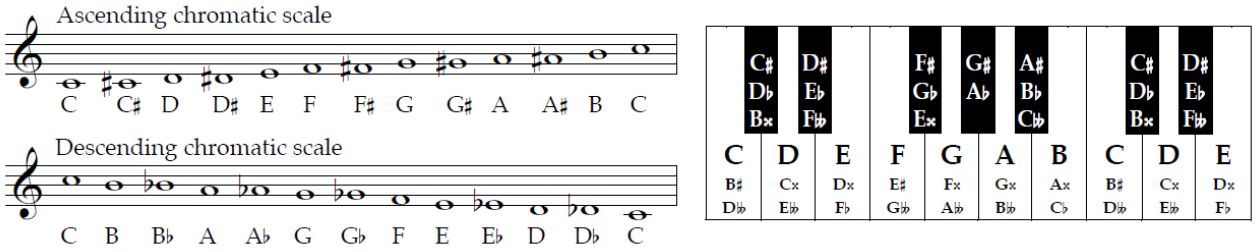
[**Figure 4.2**: Intervals between the diatonic/major scale's degrees and the tonic.]

Observe that moving a diatonic degree a fifth higher or a fourth lower produces the same note in different octaves (e.g., G1 and G2 are respectively a fourth below and a fifth above C2), and the same applies to the other couples of intervals: third-sixth, second-seventh, prime-octave.

However, the names "prime", "second", "third", … are not sufficient for completely characterizing the intervals: e.g., both F-A and E-G are third intervals, since they involve three degrees, but they correspond to a different number of semitones. Therefore, the subdivision of the octave in 12 semitones is needed for properly classifying the intervals. The scale that is produced from the equal-tempered subdivision of the octave in 12 half-steps is called **chromatic scale**:

C, C#/Db, **D**, D#/Eb, **E**, **F**, F#/Gb, **G**, G#/Ab , **A** , A#/Bb, **B**, (**C**)

where the bolded letters mark the diatonic scale, and the non-diatonic notes can be written either using a sharp or a flat. To be specific, as will be discussed later, the ascending form of the scale is written with sharps, and the descending with flats, as shown in figure 4.3.



[**Figure 4.3**: The chromatic scale.]

The chromatic scale allows to transport the intervals higher or lower without changing the number of semitones they comprehend. E.g., the third interval F-A can be written one semitone higher: F#-A#, or Gb-Bb. Therefore, the chromatic scale permits the **transposition**: a group of notes (melodies or chords) is said to be *transposed* when all the notes are written the same interval higher or lower than their original pitch. In particular, the chromatic scale allows to write the diatonic scale at different pitches: after the transposition, some notes will have acquired or lost accidentals for preserving the nature of the diatonic scale. These accidentals are written in the key signature, as it will be discussed in chapter 6.

The presence of 12 notes in the chromatic scale produces 12 intervals (in the range of the octave):

- **unison** (or *prime*) [0 ¢]: the distance between a note and itself.
- **minor second** [100 ¢]: the semitone.
- **major second** [200 ¢]: e.g., the tonic-supertonic interval in both the major and the natural minor scales. It corresponds to the tone.
- **minor third** [300 ¢]: e.g., the tonic-mediant interval in the natural minor scale.
- **major third** [400 ¢]: e.g., the tonic-mediant interval in the major scale.
- **perfect fourth** [500 ¢]: e.g., the tonic-subdominant interval in both the major and the natural minor scales.
- **tritone** [600 ¢]: the interval between the subdominant and the leading-tone in the major scale, or the supertonic-submediant interval in the natural minor scale.
- **perfect fifth** [700 ¢]: e.g., the tonic-dominant interval in both the major and the natural minor scales.
- **minor sixth** [800 ¢]: e.g., the tonic-submediant interval in the natural minor scale.
- **major sixth** [900 ¢]: e.g., the tonic-submediant interval in the major scale.
- **minor seventh** [1000 ¢]: e.g., the tonic-subtonic interval in the natural minor scale.
- **major seventh** [1100 ¢]: e.g., the tonic-leading-tone interval in the major scale.
- **octave** [1200 ¢]: the total length of the scale.

Placing an accidental against a note of the diatonic scale produces a **chromatic alteration**, since the note which is obtained belongs to the chromatic scale only. If a sharp (or a natural sign that cancels a flat of the key signature) is used, the note is said to be **chromatically raised**, if a flat (or a natural sign that cancels a sharp of the key signature) is used, the note is **chromatically lowered**. In both the cases, the note is said to be *chromatically altered*.

By chromatically altering either the upper or the lower note of the above intervals, the following happens:

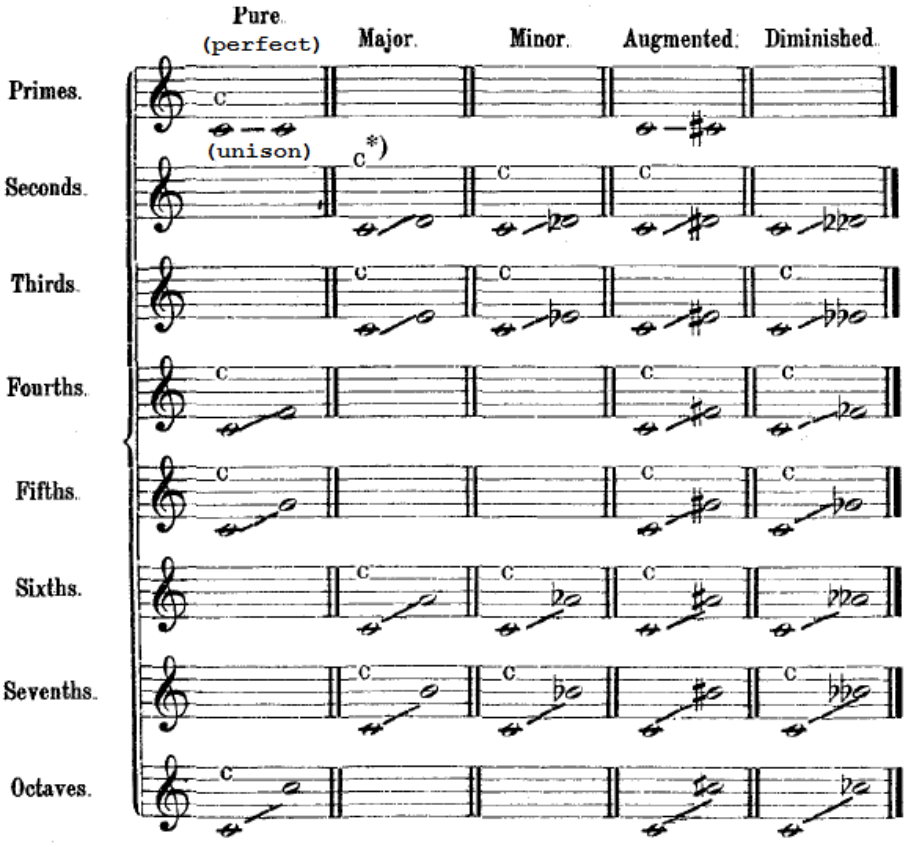
- If a minor interval is enlarged by chromatically altering one of its notes (i.e., the upper note is chromatically raised, or the lower note is chromatically lowered), then it becomes a major interval. E.g., the minor third E-G becomes either the E-G# or the Eb-G major third.
- If a major interval is reduced by chromatically altering one of its notes, then it becomes a minor interval. E.g., the major sixth C-A becomes either the C#-A or the C-Ab minor sixth.
- If a perfect or a major interval is enlarged by chromatically altering one of its notes, then it is called **augmented interval**. E.g., the perfect fifth D-A becomes either the D-A# or the Db-A augmented fifth; the major third G-B becomes either the G-B# or the Gb-B augmented third.
- If a perfect or minor interval is reduced by chromatically altering one of its notes, then it is called **diminished interval**. E.g., the perfect fifth D-A becomes either the D#-A or the D-Ab diminished fifth; the minor third A-C becomes either the A#-C or the A-Cb diminished third.

Every interval that includes a chromatically altered note preserves the name (prime, second, third, …) of the interval without the chromatic alteration. E.g., the interval G-Bb is a minor third, because the involved diatonic degrees are G, A and B; but the interval G-A#, which has the same amplitude of G-Bb, is an augmented second, because the diatonic degrees are only G and A.

The same applies to the intervals where both the upper and the lower notes have been chromatically altered: in such a case, a minor interval can become augmented, and a major interval can become diminished. E.g., transforming the minor sixth A-F into Ab-F# produces an augmented sixth; transforming the major third F-A into F#-Ab produces a diminished third.

The tritone between F and B is the only interval that doesn't need accidentals for being either an augmented fourth (F-B) or a diminished fifth (B-F). Observe that F and B are the only notes of the C diatonic scale which are separated by a tritone interval.

In the figure 4.4 the full list of the intervals is shown.



[**Figure 4.4**: All the intervals in the range of the octave.]

It is also to be noted that the augmented and diminished intervals can have the same amplitude of a major, minor or perfect interval: the labels "augmented" and "diminished" only indicate that an interval has been produced by a chromatic alteration (except the special case of the tritones F-B and B-F).

The addition of an octave to the above listed intervals produces the ninth interval (second+octave), the tenth interval (third+octave), the eleventh interval (fourth+octave), and so on. These intervals are very similar to the original ones, since the octave-extension doesn't change their main acoustic qualities. Also their names continue to be major/minor, augmented/diminished or perfect: e.g., C2-E3 is a major tenth, C2-G3 a perfect twelfth, C2-F#3 an augmented eleventh.

Transporting the lower note of an interval to the octave above, as well as transporting the higher note to the octave below, produces an **inversion** of the interval. That is, an inverted interval has its notes interchanged: e.g., G-C is the inversion of C-G. The inversions of all the intervals but the tritone (which divides the octave in two equal parts) have different amplitudes than the original interval. To be specific:

- The inversion of the unison is the octave.
- The inversion of the octave is the unison.
- The inversion of the minor second is the major seventh.
- The inversion of the major second is the minor seventh.
- The inversion of the minor third is the major sixth.
- The inversion of the major third is the minor sixth.
- The inversion of the perfect fourth is the perfect fifth.
- The tritone remains the tritone. To be specific, the inversion of the augmented fourth is the diminished fifth, and the inversion of the diminished fifth is the augmented fourth.
- The inversion of the perfect fifth is the perfect fourth.
- The inversion of the minor sixth is the major third.
- The inversion of the major sixth is the minor third.
- The inversion of the minor seventh is the major second.
- The inversion of the major seventh is the minor second.

Observe that the inverted major intervals are minor and the inverted minor intervals are major; the inverted diminished intervals are augmented and the inverted augmented intervals are diminished; the inverted perfect intervals remain perfect.

Couples of notes that have some of their initial harmonics in common often form a **consonant interval**. To be specific, in the range of the octave the consonant intervals are:

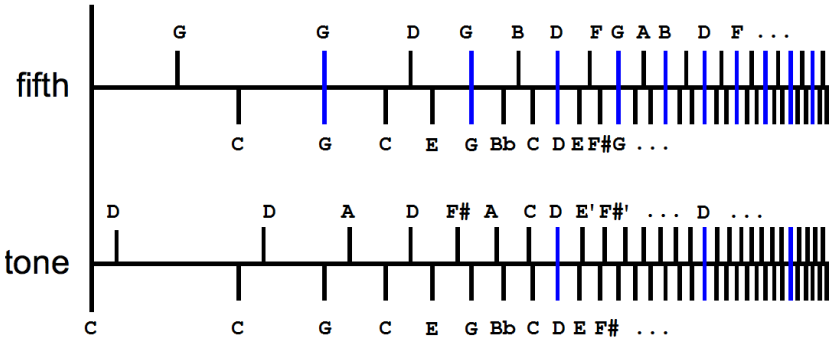
- Octave and unison
- Perfect fifth
- Perfect fourth (not always)
- Major and minor thirds
- Major and minor sixths

The remaining ones, the tritone, the (major and minor) seconds, and the (major and minor) sevenths are **dissonant interval**s.

By looking at the list of the inversions of the intervals, it can be noted that the inversions of the consonant intervals are consonant, and the inversions of the dissonant intervals are dissonant.

As discussed in chapter 1, the human brain recognizes the intervals by comparing the harmonics of the two involved notes. The consonance or dissonance of an interval is related to the amount of computation that is required by the brain for its identification: as the computation becomes easier, the consonance of the interval becomes greater, and vice-versa. E.g., the octave is the easiest interval to be recognized because the two notes have exactly the same harmonics, and therefore it is the most consonant interval.

The term "dissonant", anyway, it's not related to a bad or fastidious sound: on the contrary, for an interval or a chord to be dissonant means exclusively that it creates tension, that is, it generates in the listener the need for another interval or chord to follow. From an acoustic point of view, the sound of the dissonant intervals has inequalities called "beats". In chapter 9 the acoustic aspects of the consonance or dissonance of the intervals will be discussed.



[**Figure 4.5**: Harmonics of two notes a perfect fifth apart (on the above), compared with the harmonics of two notes a major second apart (on the below). The blue lines mark the harmonics that the upper and the lower notes have in common.]

Among the consonant intervals, the unison, the octave, the perfect fifth and the perfect fourth are also called *perfect consonances*, whereas the (major and minor) thirds and the (major and minor) sixths are *imperfect consonances*. Actually, the term "perfect consonance" don't be totally appropriate in the case of the perfect fourth interval: in fact, when the fourth is placed right above the lowest note of a chord (or when writing for two voices or instruments only), it may sounds dissonant. On the contrary, if the fourth separates two of the upper notes of a chord, it is consonant. This happens because the fourth's upper note doesn't belong to the initial harmonics of the lower note.

The diatonic scale comprehends a great amount of consonant intervals. In fact, there are six fifth intervals (C-G, D-A, E-B, F-C, G-D, A-E), three major thirds (C-E, F-A, G-B), four minor thirds (D-F, E-G, A-C, B-D), and respectively the same number of fourths (the fifth intervals inverted), minor sixths (the major thirds intervals inverted) and major sixths (the minor thirds intervals inverted). In addition, the whole length of the scale is the octave.

All the consonant and dissonant intervals can be found by looking at the distances between the harmonics. In particular, the consonant ones involve the first 8 harmonics, as it was anticipated in chapter 1. In fact, the octave is the distance between the 1st and the 2nd harmonics, the perfect fifth is the distance between the 2nd and the 3rd, then follows the perfect fourth (3rd-4th), the major third (4th-5th), and the minor third (5th-6th). Moreover, the distance between the 3rd and the 5th harmonics is a major sixth, and the distance between the 5th and the 8th is a minor sixth.

Consequently, it is common to identify the intervals, instead of taking a couple of notes, by specifying the corresponding distance between the harmonics. To be specific, being multiples of the fundamental frequency, the harmonics have the property that the 1st harmonic is half the frequency of the 2nd, the 2nd is 2/3 the frequency of the 3rd, the 3rd is 3/4 the frequency of the 4th, and so on. This relationship can be used for calling the intervals by their **frequency ratio**: the interval between the 1st and the 2nd harmonics (the octave) corresponds to the 1:2 ratio, the interval between the 2nd and the 3rd harmonics (the fifth) corresponds to the 2:3 ratio, and so on. In general, the interval between the harmonic number *n* and the harmonic number *m* corresponds to the frequency ratio *n:m*.

5-Building the diatonic scale

The music of almost every culture has evolved around particular scales. The shape of the scales has been influenced by uncountable factors, such as the human perception of the involved intervals and the technological advancement of the adopted instruments. Thus, since a scale is the result of some human-dependent aesthetic and historical development, a purely acoustic analysis cannot completely explain its origins.

However, the acoustic phenomena being involved in the particular form of a scale contribute in explaining some aspects of the music that from the scale is produced. Moreover, such phenomena are related to the brain's capability of computing the relationships between the scale's notes, and thus they can justify part of the processes that led to the preference of the scale over its variants that have appeared and then have became obsolete.

The procedure which is usually adopted in the "scientific" derivation of a scale consists of considering some of its degrees and then, by looking at their first harmonics, to justify the presence of the other degrees. Moreover, in addition to the harmonics, also the harmonics of the harmonics, called **secondary harmonics**, are taken into consideration. For being distinguished from the secondary harmonics, the original ones are also called *primary harmonics* or *generators*.

Not all the first harmonics are available: some of them, in particular the 11^{th} and 13^{th}, are considerably out of tune, being their equal-temperament approximation quite large (respectively -49 ¢ and +31 ¢). Then, they cannot be used as secondary harmonics as well.

A frequently adopted derivation of the diatonic scale consists of considering the harmonics of the tonic, the dominant and the subdominant degrees. That is, in the case of the C diatonic scale, C, G and F. As explained e.g. in text *Theory of Harmony* by A. Schonberg, this choice is driven by the fact that G, the 3^{rd} harmonic of C, brings the strongest contribution to C because it is the first harmonic of C that is different from the octaves. Therefore, its harmonics are the most significant secondary harmonics in the sound of C. Furthermore, the harmonics of C can themselves be viewed as secondary harmonics: since C is the 3^{rd} harmonic of F, its harmonics are the most significant secondary harmonics of F.

Thus, let's consider the first 5 harmonics C, G and F:

C, C, G, **C**, **E**, …

G, G, **D**, **G**, **B**, …

F, F, C, **F**, **A**, …

where the bolded diatonic scale's notes lie in the same octave.

For displacing them in the right order, one method is to directly look at their frequencies. However, the analysis can provide deeper results whenever a greater number of harmonics is taken into

consideration. Let's display the harmonics of C, F and G so that the 9th harmonic of F is aligned with both the 6th harmonic of C and the 4th harmonic of G (they all are the note G). Leaving empty some positions, one gets:

F: F F C F _ A **C** _ **Eb F G A** …

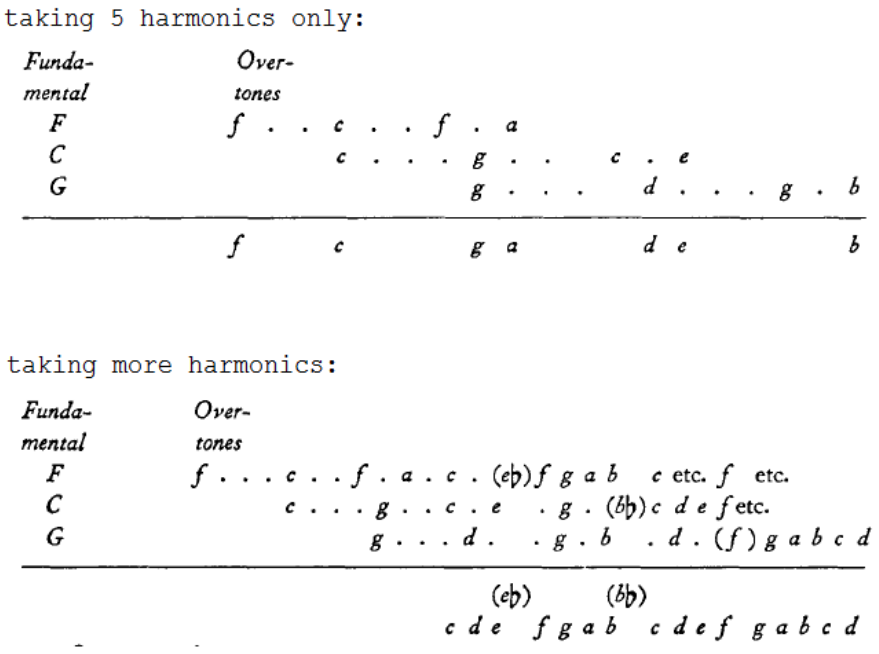
C: C _ C _ G _ **C** _ **E** _ **G** _ **Bb** C …

G: G _ _ _ G _ _ **D** _ _ **G** _ **B** _ _ D…

Then, look at the bolded letters. In all the three sequences:

- The 7th position is occupied by C (or it has been left empty)
- The 8th position is occupied by D (or it has been left empty)
- The 9th position is occupied by either E or Eb (or it has been left empty)
- The 10th position is occupied by F (or it has been left empty)
- The 11th position is occupied by G in all the sequences
- The 12th position is occupied by A (or it has been left empty)
- The 13th position is occupied by either B or Bb (or it has been left empty)

Observe the presence of both B and Bb: if proceeding with the following octave, where the harmonics of G have greater volumes, B is favored over Bb. Also, E is to be preferred in place of Eb because it's the 5th harmonic of C (that is, a primary harmonic of the tonic).



[**Figure 5.1**: Harmonics of F, C and G (graphics by A. Schonberg).]

Some authors justify the diatonic degrees by considering, in place of the subdominant, the supertonic (in the C diatonic scale, D). With such selection, the scale is derived as follows:

- Tonic
- Supertonic
- Mediant: 5th harmonic of the tonic
- Subdominant: 7th harmonic of the dominant (secondary harmonic)
- Dominant
- Submediant: 3rd harmonic of the supertonic (secondary harmonic)
- Leading-tone: 5th harmonic of the dominant (secondary harmonic)

The two analysis don't be in opposition, and the aspects in which they differ provide further elements in comprehending the importance that the diatonic scale has acquired during the centuries.

6-Tonality and triads

Whereas a melody is related to the horizontal dimension of the music, and thus it mainly belongs to the interests of counterpoint, a superposition of different melodies produces the simultaneous manifestation of different notes, that is, it involves the vertical dimension. The vertical aspect of the music, the harmony, is so important that, since the beginning of the common practice era, the melodies most often originate as a consequence of a previously established chords-progression, which assumes the function of compositional starting point.

The notes that are listened along with the leading melody, in fact, have a crucial influence on the perception of the melody itself: the melody's impact on the listener mainly depends on them. The principles of harmony, thus, involve the way in which the human brain perceives such superposition of notes. The brain computes the relationships between the notes, and then provides pleasant or unpleasant sensations.

In particular, the brain detects the harmonics of the notes. As noted in chapter 1, it doesn't have the ability to recognize the notes' frequencies (except in the people that have the absolute pitch): it only recognizes the intervals. The brain compares the harmonics of the two notes that form the interval, and after that it identifies the interval. E.g., if the brain perceives that the harmonics of two different notes are coincident, it identifies the octave interval; if it perceives that the 2^{nd}, 4^{th}, 6^{th}, … harmonics of the upper note coincide with the 3^{rd}, 6^{th}, 9^{th}, … harmonics of the lower note, it recognizes the perfect fifth interval. On the contrary, whenever the brain finds a small quantity of common harmonics (among the initial ones), its elaboration becomes difficult: this produces the effect of dissonance.

Moreover, if the fundamental frequency is artificially subtracted from the harmonics of a note, it has been shown that the listener will continue to detect the same note, that is, he will continue to hear the non-existing 1^{st} harmonic. The brain processes the other harmonics, and spontaneously calculates the fundamental frequency. This phenomenon is called **virtual pitch**, and it is used in engineering e.g. when the speakers are too physically small to produce the lowest notes: instead they play only their harmonics, and rely on the brain's computation to reconstruct the note.

This suggests that, when a melody or a chord adopts notes that coincide with the harmonics of a fundamental frequency, or with its secondary harmonics (see the previous chapter), such fundamental frequency is to some extent required by the listener. Under the light of the virtual pitch, the acoustic affinity of the diatonic scale's tonic with the other degrees contributes to recognize the tonic as fundamental frequency, and to perceive the other notes in their relation with it. Consequently, during the time in which a melody or a chord doesn't comprehend the tonic, a more or less strong sense of instability and tension is produced. Then, when the tonic is finally sounded, it is felt as a resting point and provides a pleasant sense of conclusion.

In other words, the tonic pulls towards itself all the other notes: this is the concept of **tonality**. The tonality is the relationship between all the notes of a piece of music and one particular note, the

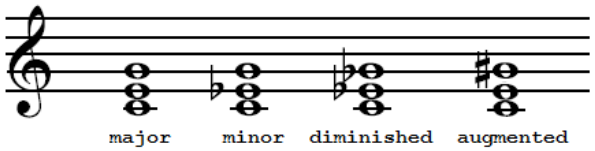
tonic of the scale which is used. This relationship is established by the harmonics (and secondary harmonics) that the tonic has in common with the other degrees of its scale.

A tonality is called **major tonality** (also **major key** or **major mode**) if the major scale is adopted, and it's called **minor tonality** (**minor key**, or **minor mode**) if the natural minor scale (and its two variants, as discussed in the next chapter) is adopted. Since the notes of the major and the natural minor scales are the same, the establishment of either a major or a minor tonality requires that the primary role of the corresponding tonic degree (and its first harmonics) is emphasized by the composer. The tonic of the major or the natural minor scale is therefore called *tonic of the key* or *root of the key*. The **tonal music** consists of the music where a major or minor tonality is recognizable.

A displacement of three notes whose relative distance is a major or minor third interval is called **triad**. From this definition, it follows that four types of triad are available:

- The **major triad**, where a major third lies between the lowest note and the middle note, and a minor third is between the middle and the upper note. E.g., C-E-G is a major triad.
- The **minor triad**, where a minor third lies between the lowest and the middle note, and a major third is between the middle and the upper note. E.g., C-Eb-G is a minor triad.
- The **diminished triad** is made up by adding two minor thirds. E.g., C-Eb-Gb is a diminished triad.
- The **augmented triad** is made up by adding two major thirds. E.g., C-E-G# is an augmented triad.

The lowest note (the bass) is called the **root** of the triad (the term "root" is also used for the bass of the other chords). The middle note is called "third" of triad and the upper note is the "fifth", because they respectively form a third interval and a fifth interval with the root. The four types of triad are shown in figure 6.1.



[**Figure 6.1**: The four types of triad.]

The root is the most important element of a triad. When the root of a major triad is the tonic of the major scale, and when the root of a minor triad is the tonic of the natural minor scale, the triad is called **tonic triad**. E.g., the tonic triad of the C major tonality (where the tonic is C) is the major triad C-E-G, and the tonic triad of the A minor tonality (where the tonic is A) is A-C-E.

The tonic triad is the most important device for establishing the tonality in piece of music. It is the sound that the listener recognizes as the main point of arrival where the musical phrases should end. In other words, this triad provides a sense of conclusion every time the music returns to it. As

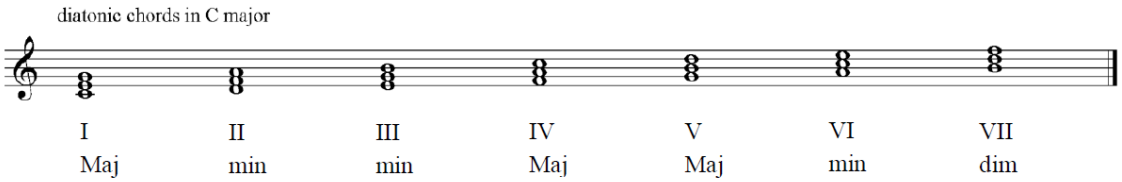
a consequence, is very frequent to have the tonic triad at the beginning and at the end of a piece of music.

The fundamental role that the tonic triad plays in tonal music is related to the acoustic aspects of the major and minor triads. To be specific, the major triad is the best ambassador of the harmonics of its root: taking e.g. the C-E-G major triad, the third (E) and the fifth (G) coincide with the first two harmonics of the root (C) which are different from its octaves. Thus, the major triad omits the more distant harmonics of its lowest note and reinforces the more immediate. Also the minor triad has analogous properties, and will be analyzed in the next chapter.

In the music of the common practice period, a large amount of chords are either major or minor triads, or can be interpreted as such. In particular, each degree of the diatonic scale can become the root of either a major, a minor or a diminished triad whose third and fifth correspond to two of the other degrees. To be explicit, let's consider the case of C major tonality. It "naturally" includes, since chromatic altered notes don't be used, the following triads (shown in figure 6.2):

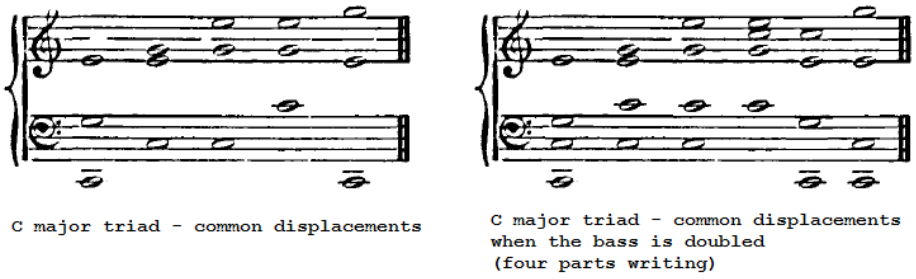
C-E-G ; **D**-F-A ; **E**-G-B ; **F**-A-C ; **G**-B-D ; **A**-C-E ; **B**-D-F

These chords are known as **diatonic triad**s. The triads C-E-G, F-A-C and G-B-D are major triads; D-F-A, E-G-B and A-C-E are minor triads; and B-D-F is a diminished triad. These triads also belong to the A minor tonality, since the diatonic scale is the same.



[**Figure 6.2**: The diatonic triads.]

Triads can appear in different forms. In fact, by transporting the third, or both the two upper notes, to an octave higher, then the triad's acoustic features and qualities don't change. Also, by duplicating any of its notes (more often the bass or the perfect fifth) onto the octaves above, the nature of the chord remains the same: it is in fact the ordinary practice when writing for more than three elements. In all these cases, the root remains the lowest note.



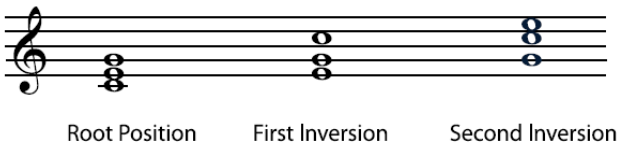
[**Figure 6.3**: Examples of C major triad's displacement in three and four parts writing.]

In the figure 6.3 some common solutions for placing the C major triad on the grand staff are shown, both in the case of three and four parts writing (in this case, the bass has been doubled). Not all the other possible displacements are equally good, as it will be discussed in chapter 10.

If the displacement of the triad's notes is altered so that the root doesn't be the bass, quite different chords arise: they are called **triad inversion**s, because involve the inversions of the intervals that the third and the fifth form with the root. To be specific, triads have 2 inversions (shown in figure 6.4):

- The chord which is obtained by putting the root on the octave above (or by putting the upper two notes on the octave below) is called **first inversion**. E.g., the first inversion of C-E-G is E-G-C, where the bass is E.
- The chord which is obtained by putting the lower two notes on the octave above (or by putting the fifth on the octave below) is called **second inversion**. E.g., the second inversion of C-E-G is G-C-E, where the bass is G.

Thus, the first inversion places the third in the bass, the second inversion does the same with the fifth. Observe that the inversion of an augmented triad produces another augmented triad.



[**Figure 6.4**: Triad inversions.]

The first inversion is also known as *sixth chord*, or *chord of the third and sixth*, because the distances between the bass and the other two notes are respectively a third and a sixth interval. Similarly, the second inversion is called *six-four chord*, or *chord of the fourth and sixth*, because a fourth and a sixth intervals are formed with the bass. If a triad doesn't be inverted, is said to be in **root position**, because the root is the bass. When speaking of "C major triad", the letter "C" always refers to the root of the triad, even if the triad is in inversion.

The distinction between the root position and its inversions is related to the primary role that the bass assumes in the chords. The influence of the bass on the notes above it, in fact, is a central element of tonal music: being the lowest note, the harmonics of the bass predominate over those of the other notes. The pleasantness or solidity of the chords, therefore, depends on the affinity of the upper notes with the lowest one; that is, it depends on the amount of harmonics that the upper notes have in common with the bass. As discussed in chapter 8, this is the reason because the inversions of the major and minor triads sound weaker and more undecided than their root position.

Since the inversions don't change the consonant/dissonant character of the intervals, chords are very often replaced with their inversions. However, being less incisive, inversions are seldom used for concluding the musical phrases.

Because each diatonic degree is the root of only one type of triad (major, minor or diminished), it is frequent to indicate the diatonic triads by adopting the Roman numerals I, II, III, IV, V, VI, VII: the Roman numerals specify the corresponding scale's degree upon which the triads are built, that is, they specify their root. E.g., the passage from the triad whose root is the fifth degree (V) to the tonic's triad (I) is written as V-I.

There are different styles for writing the numerals: writers in German use only upper-case letters, writers in English often adopt upper-case letters for major and augmented triads, and lower-case letters for minor and diminished triads. Moreover, there is a quantity of additional signs for indicating e.g. if the triad is in root position or in inversion, if additional notes are placed, if the triad is subjected to chromatic alterations, and so on. In the following, only the upper-case Roman numerals will be used.

Behind the tonic triad, there are two other important chords: the triad where the root is the dominant degree (i.e., V) is called **dominant triad**, and the triad where the root is the subdominant degree (IV) is the **subdominant triad**. These triads also serves well as temporary points of arrival. Observe that IV and V lie respectively a fourth and a fifth above I (that is, a fifth above and a fifth below), so that their roots form perfect consonances with the tonic. These two triads, along with the tonic triad, are to be considered the main reference sounds of the tonality. Observe that, if taken together, I, IV and V include all the diatonic degrees (both in major and minor tonalities): it would be possible to harmonize whatever melody on the diatonic scale using these three triads only.

Moreover, in the previous chapter a derivation of the C diatonic scale was obtained by taking the harmonics of the fifth degree above and the fifth degree below the tonic (G and F). In particular, the harmonics of the tonic C were regarded as secondary harmonics of F (whose fifth is C). This point of view is related to the essential contribution that the 3rd harmonic and its secondary harmonics provide to the fundamental frequency. This contribution is well-perceived by the brain, which always tries to recognize a harmonics sequence among the notes that are listened: the brain tends to consider the harmonics of G as secondary harmonics of C, even if C is not present. As a consequence, it is possible to identify in G a significant predisposition to move towards the tonic C.

More generally, any note behaves like a dominant degree, since any note can always be thought as the fifth above a tonic, i.e. its harmonics can always be considered secondary harmonics. The same applies to the triads as well: every triad, especially if major, has the tendency to fall on the fifth below its root through a progression of the type V-I. The motion V-I, that will be discussed in chapter 15, is the strongest progression that it's possible to make by using two triads only.

Being the most incisive connection to I, the progression V-I reinforce the listener's perception of I as the tonic triad. Therefore, it is the most adopted device for introducing in the music a new tonic

triad, that is, a new tonality. A change of the tonality in the middle of a piece is called a **modulation**. The modulations, that will be deepened in chapter 16, are of very frequent usage in the common practice music, especially the Romantic period. Since a change of the tonality corresponds to the adoption of a different diatonic scale, a different key signature is required.

The key signature comprehends the set of accidentals which are incorporated in the diatonic scale that is used in a piece of tonal music. The couples of (major and minor) tonalities that have in common the same diatonic scale (e.g., C major and A minor), therefore, have identical key signatures. Such couples are sometimes called "twins tonalities"; in particular, the minor tonality is called **parallel minor** or **relative minor** of the major tonality (and, vice-versa, the major tonality is called **parallel major** or **relative major** of the minor tonality). By looking at the key signature, thus, it's impossible to understand if either the major or the minor tonality is used, and the analysis of the score is needed.

Since the C diatonic scale doesn't have accidentals, no accidentals are written in the key signature of C major and A minor tonalities. The key signature of the other tonalities is obtained by transposing the C diatonic scale to higher or lower pitches. Since there are 12 semitones in one octave, 12 transpositions are possible. And since each diatonic scale provides two tonalities, 24 tonalities are available.

E.g., the G major key (and its parallel minor, E minor) has one accidental, F#, in the key signature, because in the diatonic scale that begins on G:

G, A, B, C, D, E, F#

a sharp on the seventh degree is present. To write Gb would be wrong, because the leading-tone forms a major seventh with the tonic G, and not a diminished octave.

Similarly, the transposition of the C diatonic scale to F produces:

F, G, A, Bb, C, D, E

where Bb allows to obtain the diatonic's fourth degree (writing A# would produce an augmented third above the tonic, and not the perfect fourth). Consequently, F major and D minor have Bb in the key signature.

The transposition to the remaining diatonic degrees, D, E, A, and B, follows in analogous manner (as the reader can check, they all involve only sharps). In the case of the non-diatonic degrees of the chromatic scale, the corresponding diatonic scale can be written either using only sharps (and, eventually, double-sharps) or only flats (or double-flats). E.g., the diatonic scale of Db major (Bb minor) is:

Db, Eb, F, Gb, Ab, Bb, C

and the diatonic scale of C# major (A# minor) is:

C#, D#, E#, F#, G#, A#, B#

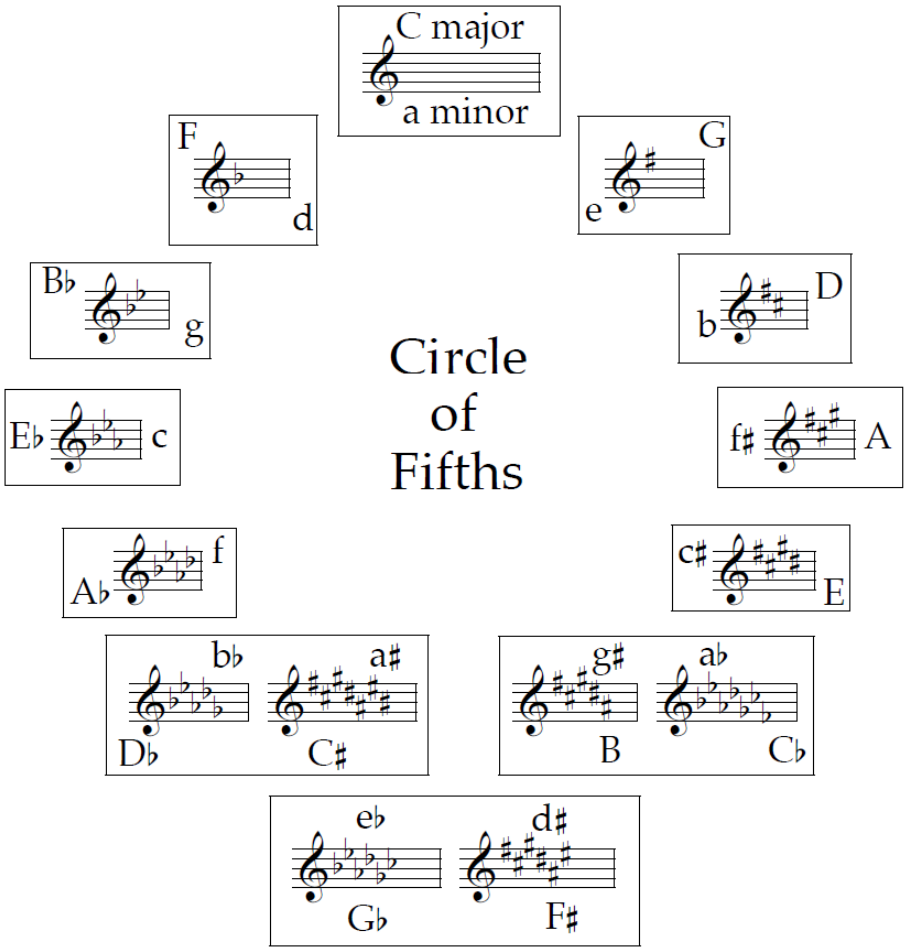
In the choice between C# and Db, D# and Eb, F# and Gb, G# and Ab, A# and Bb, the smaller number of accidentals which are produced in the transposition leads to prefer Db over C#, Eb over F#, Ab over G# and Bb over A#.

From the transposition of the diatonic scale, a general criterion can be recognized: taking the C diatonic scale as the starting point, the key signature acquires a sharp when transposing to the fifths above (G, D, E, B, F#) and acquires a flat when transposing the fifths below (F, Bb, Eb, Ab, Db, Gb). That is:

- The transposition to the fifth above C major (G major) produces F#, and all the other keys involving sharps also have F# in the key signature. The transposition to the fifth above G major (D major) produces, in addition to F#, the note C#, and all the remaining keys involving sharps also have both F# and C#; and so on.
- The transposition to the fifth below C major (F major) produces Bb, and all the other keys involving flats also have Bb in the key signature. The transposition to the fifth below F major (Bb major) produces, in addition to Bb, the note Eb, and all the remaining keys involving flats also have both Bb and Eb; and so on.

Therefore, while varying in number for differentiating the key signature, the accidentals follow a fixed sequence in their coming out. In the case of the sharps, the sequence is: F#, C#, G#, D#, A#, E#, B#. If a tonality comprehends e.g. A#, then all the preceding sharps in the list (F#, C#, G#, D#) are also present in its key signature. The same applies to the flats, but the order of appearance is reversed, that is, they follow the opposite sequence: Bb, Eb, Ab, Db, Gb, Cb, Fb. If a tonality comprehends e.g. Gb, then all the preceding flats in the list (Bb, Eb, Ab, Db) are also present in its key signature.

The keys that don't involve double-sharps and double-flats (i.e., the most frequently used ones) can be displaced in a circular relationship, called **circle of the fifths**, which is shown in figure 6.5.



[**Figure 6.5**: Circle of the fifths.]

7-Minor tonality

The essential difference between the major and the minor tonalities is the quality of the third above the tonic: as previously noted, the tonic-mediant interval changes its amplitude whenever either the major or the natural minor scale is considered. In the major scale it is a major third (e.g., in C major, C-E), in the minor scale it is a minor third (in A minor, A-C).

The minor tonality is one of the most interesting elements of Western music: in fact, whereas the mediant degree of the major scale coincides with the 5^{th} harmonic of the tonic (E is the 5^{th} harmonic of C), the mediant degree of the minor scale (in C minor, Eb) doesn't appear neither between the initial harmonics of the tonic, nor between its initial secondary harmonics. Mostly because of this, many authors have considered the minor triad (and the minor tonality that it represents) artificial, in agreement with the Helmholtz's point of view, which regards the minor triad as "inferior" to the major triad. However, in the musical practice, the minor tonalities are diffused and appreciated as well as the major tonalities: the minor triad doesn't be perceived less consonant than the major triad, it simply sounds different.

The analysis of the harmonics involved in the minor triad may help to clarify this point. Consider the C minor tonic triad, C-Eb-G. The 3^{rd} and the 5^{th} harmonics of Eb are respectively Bb and G: Bb coincides with the 7^{th} harmonic of C, and G coincides with both the 3^{rd} harmonic of C and the 1^{st} (2^{nd}, 4^{th}, 8^{th}, ...) harmonic(s) of G. Therefore, the introduction of Eb in the sound of C and G is sustained by the two most important harmonics of Eb after its octaves.

Moreover, looking at the C natural minor scale:

C, D, Eb, F, G, Ab, Bb, (C)

it is possible to observe that both Bb and Ab (which don't be included in the C major scale) give further emphasis to Eb: in fact, Bb is the 3^{rd} harmonic of Eb, and Eb is the 3^{rd} harmonic of Ab.

It is to be noted, however, that E, being the 5^{th} harmonic of C, inevitably provides a relevant contribution to every triad whose root is C. Consequently, this harmonic may be in conflict with the presence of Eb in the C minor triad, especially when the root is doubled. This acoustic impurity, perhaps, gives to the minor mode its characteristic flavor.

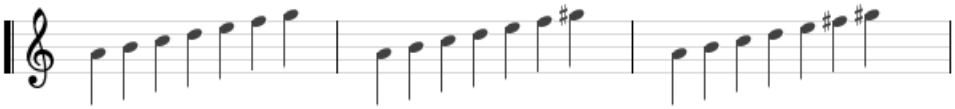
A typical aspect of the minor tonalities consists of chromatically rising the seventh degree, or both the seventh and the sixth degrees, of the natural minor scale, so that two additional scales arise. They are the **harmonic minor scale**:

A, B, C, D, E, F, G#

and the **melodic minor scale**:

A, B, C, D, E, F#, G#

which are shown in figure 7.1.



[**Figure 7.1**: The minor scales: natural minor scale, harmonic minor scale, and melodic minor scale.]

Since F# and G# have fewer harmonics in common with the tonic triad A-C-E than F and G, these chromatic changes mainly have melodic purposes, as it will be discussed in chapter 11. To be specific, G# assumes an "harmonic function" only in particular situations (as shown in chapter 15).

The introduction of F# and G# leads to the birth of new triads, which enlarge the collection that was found on the diatonic scale degrees in the previous chapter. In the case of the harmonic minor scale, the following triads arise:

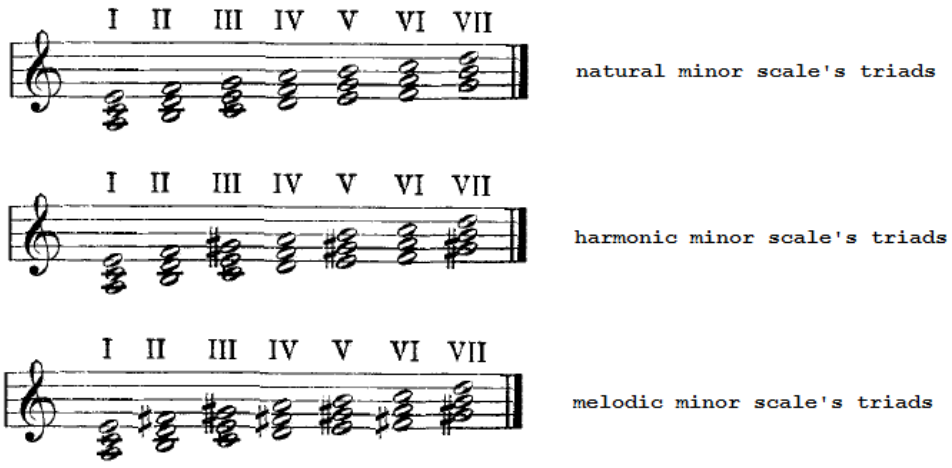
A-C-E ; **B**-D-F ; **C**-E-G# ; **D**-F-A ; **E**-G#-B ; **F**-A-C ; **G#**-B-D

The triads on the first and the fourth degrees are minor, the triads on the fifth and the sixth degrees are major, the triads on the second and seventh degrees are diminished, and the triad on the third degree is augmented.

In the case of the melodic minor scale:

A-C-E ; **B**-D-F# ; **C**-E-G# ; **D**-F#-A ; **E**-G#-B ; **F#**-A-C ; **G#**-B-D

where the triads on the first and second degrees are minor, the triads on the fourth and fifth degrees are major, the triads on the sixth and seventh degrees are diminished, and the triad on the third degree is augmented.



[**Figure 7.2**: Triads on the minor scales.]

These scales produce the augmented triad, which is absent among those obtained from the degrees of the diatonic scale. The augmented triad divides the octave in three equal parts and has special properties, as it will be discussed in chapter 17.

8-Inversions of major and minor triads

As discussed in chapter 6, triads have two inversions: in the first inversion the third is placed in the bass, while in the second inversion it is the fifth. It was noted that the harmonics of the bass provide greater contribution to the chords than that of the other notes. Therefore, in the case of the major and minor triads, a change in the bass alters the triad's stability and solidness: the acoustic aspects that make the root position particularly solid lose much of their influence.

To be specific, whereas in the major and minor triads' root position the third and the fifth have numerous harmonics in common with the bass, if the bass changes then also the harmonics that have to be considered change. In other words, the bass forms different intervals with the other two notes (in the first inversion, a third and a sixth; in the second, a fourth and a sixth). As a result, the number of common harmonics decreases.

However, since the inversion of a consonant interval produces another consonant interval, dissonances cannot arise from the inversion of a major or minor triad (except for the fourth's special behavior: as it was noted in chapter 4, the fourth with the bass produces the effect of a dissonance). Therefore, the inversions of the major and minor triads can be considered "weaker" representatives of the corresponding root positions.

The inversions of the major and minor triads have a number of useful characteristics. In fact, the lesser affinity of the new bass with the upper notes subtracts to the bass the emphasis the its lowest position would spontaneously provide. Consequently, as the bass diminishes is presence, the melody of the higher part (which is perceived with great clearness) is highlighted and strengthens its leading role. Moreover, inversions produce variety in the chords displacement because consent to avoid the repetition of the same lowest note. From a practical point of view, they offer many additional possibilities in the design of the voice-leading, and thus allow the composer to obtain smoother chord progressions, as it will be discussed in chapter 10.

As noted above, the change in the bass implies that different harmonics have to be considered in the evaluation of the acoustic empathy between the triads notes. Let's then explicit what happens when a major triad is inverted. Taking the C major triad C-E-G, the root position involves the following harmonics:

C: C_ _ _ C_ G_ C_ E_ G …

E: _ **E**_ _ _ **E**_ _ B_ **E**_ G# …

G: _ _ **G**_ _ _ _ **G**_ _ D_ _ **G** …

where the bolded letters mark the harmonics that the upper notes have in common with the bass. Then, consider the first inversion:

E: E_ _ _ E_ B_ E_ G#_ B …

G: _ G_ _ _ G_ _ D_ G_ **B** …

C: _ _ C_ _ _ _ C_ _ G_ _ C_ **E** …

and the second inversion:

G: G_ _ _ G_ D_ G_ B_ D …

C: _ C _ _ _ C_ _ **G**_ C_ E …

E: _ _ E_ _ _ _ E_ _ **B**_ _ E …

As it can be noted from the scarcity of bolded letters, in the two inversions the bass gets lesser sustainment from the upper notes. In particular, in the case of the second inversion, the harmonics which are shared with the bass (G and B) are closer to the beginning of their sequences than those of the first inversion, and therefore they usually have greater volumes. This provides to the second inversion a slight greater affinity with the root position. However, such advantage is obscured by the presence of a fourth interval between the bass and the root.

9-Consonance and dissonance

The acoustic affinity between the two notes the form an interval is established by the harmonics that these notes have in common. As noted previously, the difficulty in the detection of an interval by the brain, which compares the harmonics of the notes, provides the consonant or dissonant character to the interval. In particular, the first harmonics of the interval's notes are the most significant, being the loudest elements.

The first 5 harmonics of a note, along with the 6th (which is the octave of the 3rd) and the 8th (the third octave of the 1st), allow to find - by looking at their relative distances - all the consonant intervals which are used in Western music. It is possible to subdivide the consonant intervals into 2 groups:

- The intervals between the fundamental frequency (or its octaves) and the first 5 harmonics. That is, the unison (the interval between the fundamental frequency and itself), the octave (between the fundamental and the 2nd harmonic), the perfect fifth (between the fundamental and the 3rd harmonic) and the major third (between the fundamental and the 5th harmonic).
- The intervals that don't comprehend the fundamental, that is, the perfect fourth (the interval between the 3rd and the 4th harmonics), the minor third (between the 5th and the 6th), the major sixth (between the 3rd and the 5th), and the minor sixth (between the 5th and the 8th).

Thus, whereas in the first group the intervals' upper note is itself one of the 5 initial harmonics of the lower note, in the second group, on the contrary, the same relationship doesn't take place. E.g., if the lower note is C, the fourth interval is C-F, the minor third is C-Eb, the major sixth is C-A, and the minor sixth is C-Ab: all their upper notes, F, Eb, A and Ab, don't belong to the initial harmonics of C.

The second group (which comprehends the inversions of the intervals in the first group, plus the minor third and the major sixth) is representative of a consonant behavior which is different, and perhaps to some extent weaker, than that of the first group. In fact, the consonance of the intervals in the second group is provided by the following relationship: either the 3rd or the 5th harmonic of the upper note coincides with either the 1st, the 3rd or the 5th harmonic of the lower note. To be specific:

- In the fourth interval C-F, the 3rd harmonic of F is C, which is the 1st harmonic of C.
- In the minor third interval C-Eb, the 5th harmonic of Eb is G, which is the 3rd harmonic of C.
- In the major sixth interval C-A, the 3rd harmonic of A is E, which is the 5th harmonic of C.
- In the minor sixth interval C-Ab, the 5th harmonic of Ab is C, which is the 1st harmonic of C.

This analysis might allow to consider the intervals in the first group "tonically consonant", because their upper note coincides with one of the initial 5 harmonics of their lower note, whereas those in the second group can be viewed as "phonically consonant", because the harmonics that the upper note has in common with the lower note are either the 3rd or the 5th, but not the 1st.

In the case of dissonant intervals, on the other hand, neither the upper note nor its first 5 harmonics coincide with one of the first 5 harmonics of the lower note (i.e., they don't belong neither to the first group nor to the second group). In fact:

- In the minor second interval (C-Db), the lower and the upper notes don't have harmonics in common (among the initial ones).
- In the major second interval (C-D), D is the 9th harmonic of C, the 7th harmonic of D is C (1rd harmonic of C), and the 9th harmonic of D is E (5th harmonic of C).
- In the tritone interval (C-F#), the 7th harmonic of F# is E (5th harmonic of C).
- In the minor seventh interval (C-Bb), Bb is the 7th harmonic of C, the 5th harmonic of Bb is D (9th harmonic of C), and the 9th harmonic of Bb is C (1st harmonic of C).
- In the major seventh interval (C-B), the lower and the upper notes don't have harmonics in common (among the initial ones).

When referring to the "initial harmonics", it would be useless to consider those beyond the 9th: the 10th and the 12th are respectively octaves of the 5th and the 3rd, the 11th and the 13th are out of tune (large equal-tempered approximations are needed), and the influence of the next ones, in any case, is greatly lessened by the decreasing of the harmonics' volumes.

This analysis, however, doesn't provide explanation to the augmented and diminished intervals. In fact, whereas the acoustic (or "absolute") consonance or dissonance of an interval depends on its amplitude, in tonal music all the notes have a strong relationship with the tonic. In other words: the notes of a consonant interval can form a dissonant interval with the tonic. Thus, the augmented and diminished intervals sound dissonant because, although their amplitude can correspond to that of a consonant interval, their notes (one or both) don't belong to the diatonic scale which is adopted. Therefore, they are in conflict with the tonality which have been established.

E.g., the intervals A-F and C-Ab have the same amplitude (a minor sixth): if they are used in C major tonality (or A minor), the first is consonant and the second is dissonant. On the contrary, when they are used in Eb major tonality (or C minor), the first is dissonant and the second is consonant. In fact, in C major, A-F is always minor sixth, while C-Ab could also be written as C-G#, which is an augmented fifth. On the contrary, in Eb major, C-Ab is always a minor sixth, because Ab is in the key signature (i.e. it arises from the transposition of the C diatonic scale).

In Western music, the dissonant intervals are fundamental ingredients. The art of writing tonal music involves the ability to create climaxes and progressions, so that the listener is continuously waiting for something to come. When a sequence of chords generates an increasing effect of tension and stress, in the listener spontaneously arises the need for some stratagem to release such energy. The elements providing tension are the dissonant intervals. Chords including dissonant

intervals are called **dissonant chord**s or **discord**s, and the word "dissonance" can refer to both dissonant intervals and dissonant chords. Since the harmonics of the bass prevail over those of the other notes, the effect of dissonance is emphasized when the dissonant interval involves the bass.

The chords involving consonant intervals only, called **consonant chord**s (substantially, major and minor triads), are the targets where the dissonance's tension is discharged and leaves the place to relaxation and stability. The musical flow should be a continuous alternating of consonance and dissonance, where the dissonant chords are often grouped into dissonant progressions. We remark that the dissonances don't correspond to bad sounds: a dissonant chord is a chord creating tension, it provides dynamics. If music involved consonant chords only, it would be static, mawkish and sloppy.

The concept of the dissonance had altered noticeably during the centuries, and the use of dissonances had greatly enlarged beyond that of the seventeenth century. In the earliest harmonic writing, vocal music was frequently performed without accompaniment, and performers sang their parts mostly at sight. Consequently, dissonances were generally avoided because they are more difficult to intone than the consonant intervals. And if dissonant intervals were present, they were placed on the weak beats of the musical meter, often as passing notes between consonant chords (see chapter 14).

As the use of dissonances began to be more frequent, special carefulness was required for to embed them in a composition, also in the case of instrumental pieces. The traditional method for placing the dissonant intervals into a piece of music consists of including one of its notes in the previous chord, where such note has to take part to a consonant interval: as a result, when going to the dissonant chord, the note creates the dissonant interval without moving. This procedure is called **preparation of the dissonance**, and the note which remains held during the transition from the consonant and the dissonant interval is said to be *prepared*.

The best note to be prepared, among the two forming the dissonant interval, will be the one which is the most unrelated to the tonality and/or to the chord that comprehends it. E.g., if the dissonant interval is created by placing one accidental, one prepares the note upon which the accidental is placed; if the dissonance is introduced by inserting an additional note upon a triad, such note will be prepared.

Since dissonances create a sense of instability, they need a target where to release the tension that they have accumulated. The definitive conclusion of a dissonant progression is called the **resolution of the dissonance**, and must be a consonant chord.

As discussed in chapter 6, in any note there is the predisposition to fall a fifth below it (or to go up a fourth, which is the same), because any note can be viewed as the 3rd harmonic of the note lying a fifth below it. That is, the harmonics of a note are perceived as both primary and secondary harmonics. Accordingly, the ideal resolution of a dissonant chord would consist of such movement to the fifth below, which is performed by the root of the dissonant chord (and not the other notes).

This is often the most incisive method for releasing the dissonance's energy, but, in general, the dissonant intervals can be resolved through every progression that ends with a consonant triad.

Among the four types of triad that can be built on the major and minor scales, two dissonant examples are present: the diminished triad and the augmented triad. They don't include the fifth interval, which is replaced by dissonant elements: the tritone lies in the diminished triad, and the augmented fifth in the augmented triad.

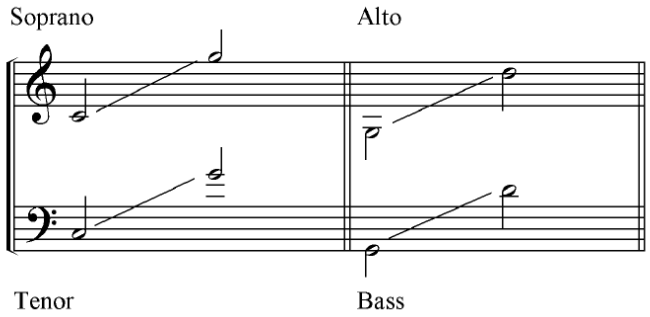
10-Traditional voice-leading

In the music of the common practice period, and, in general, in all pieces of a certain complexity, the chords are produced through the different motions of the parts. The greatest composers are able to create chords while providing each voice or instrument with an independent and pleasant melody. As the masterpieces have largely demonstrated, there are no limits to both the perfection and the complexity that different melodies sounded together can reach.

The laws for the voice-leading apply to both vocal and instrumental music. However, their historical evolution has taken place in the context of choral music. In particular, harmony was born within the most challenging environment: the choir elements usually sang at sight, without the help of instruments providing reference notes or chords. As a result, the early guidelines of the harmony theory, that focus on the four-part vocal writing, consent to obtain complex works through the superposition of simple melodies. Even if such guidelines are to a large extent neglected in the more recent works, their knowledge is the preliminary step for allowing the composer to proceed toward freer creations.

The basic implementation of the harmony's principles consists of displacing a succession of block chords, such as often occurs in the traditional four-part vocal writing, or chorale-style format. It is fundamentally the layout that was adopted e.g. by Palestrina and the other contemporaneous masters in the early times of the development of the theory of harmony, and it can be viewed as the quintessence of the harmonic practice.

The chorale-style format is notated on two staves, with each staff containing two parts: soprano and alto voices on the upper staff, tenor and bass voices on the bottom staff. Soprano and tenor are written with stems up, the others with stems down. Each part is restricted in its limited range (see figure 10.1) and the crossing between voices (one voice goes higher than one of its neighboring voices) should be avoided.



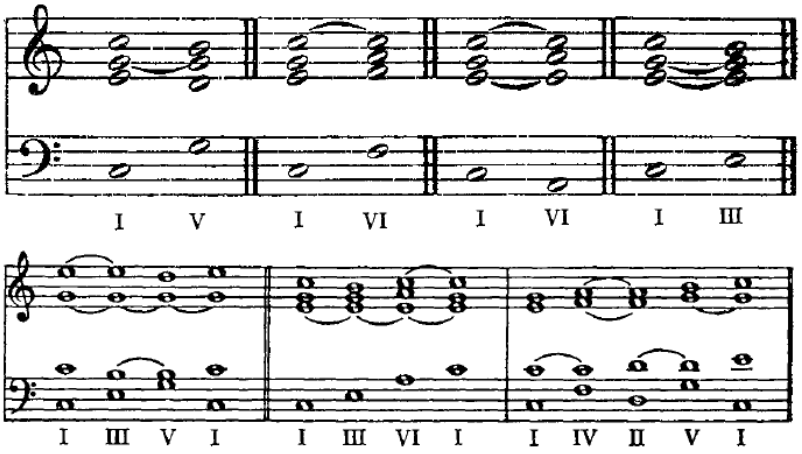
[**Figure 10.1**: Voice ranges in the traditional four-part vocal writing.]

When listening to a polyphonic work, the highest part's melody is perceived with greater clearness than the lower voices (because of the higher pitch itself). Therefore, it is to be considered the song's leading melody. Behind the higher part, also the bass is easily identified by the listener, and

thus assumes a secondary melodic function (in fact, it is sometimes indicated as the "second melody"). The highest and the lowest parts are called the *outer parts*, while the remaining ones (lying in the middle) are the *inner parts*.

As a general rule for to obtain fluent and homogeneous chords connections, the inner parts should be the most stationary ones. They should move only if they really have to, making no more than the actions which are absolutely needed for connecting the chords. In particular, in the change from one chord to the adjacent one, the notes that they have in common should be held in the same part. When a note is sustained during the transition from one chord to the following, it is called a **common tone**.

Common tones can be viewed as a very powerful glue for connecting different chords. Observe that not only the note is sustained: also its harmonics continue to live. Thus, a greater number of notes are actually shared between consecutive chords that involve common tones. E.g., the tonic's triad shares two notes with both the triad on third degree and the triad on the sixth degree; and shares one note with the triads on the fourth and on the fifth degrees. Therefore, while performing the connections I-III, I-IV, I-V and I-VI, the common tones should be sustained (see figure 10.2). The use of chords' inversions considerably enlarges the set of the available possibilities for allowing the common tones to belong to the same part.



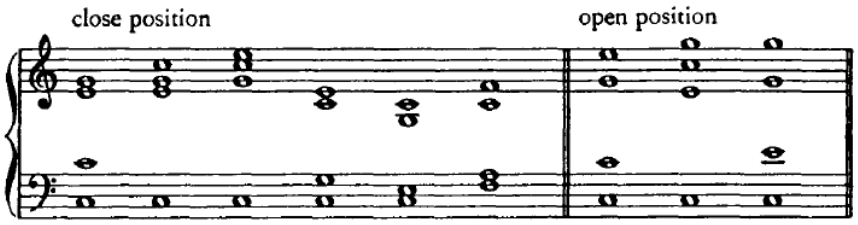
[**Figure 10.2**: Examples of common notes tying.]

Whenever no common notes are available, and either a step or half-step motion is not practicable, each part should avoid leaps through other dissonant intervals. In addition, the sum of two consecutive leaps (in the same direction) shouldn't be a dissonant interval, because the ear perceives the effect that would be obtained if the notes had sounded simultaneously. In general, the parts usually don't span more than a fifth through only one movement, and when they really have to make big jumps, a motion in the opposite direction should follow. The highest part only is allowed to freer stunts because of the aesthetic purposes that its melody is equipped with.

When writing triads for more than three parts, it arises the problem to assign a note to each part by doubling the triads' pitches, i.e. by putting the same note in two (or more than two) parts, in unison or at different octaves. In the case of the major and minor triads, the preferred note to be duplicated is the root; as a second choice, the fifth. The doubling of the middle note (both in major and minor triads) produces a weaker result, though it is very frequent. When dealing with the diminished and the augmented triads, on the other hand, the fifth (that form a dissonant interval) shouldn't be duplicated.

Although it doesn't exist a rigid criterion for displacing the notes through the vertical dimension, for obtaining a balanced sound, in general, the smallest intervals between the notes should lie sufficiently high in the staff, so that the upper voices are more often spaced closer to each other than the lower ones. In particular, the presence of the intervals smaller than the fifth in the low registers often sounds bad, even if they are consonant, especially if the upper parts are distant from the two lowest ones. This can be related to the natural displacement of the harmonics, where the higher elements are separated by smaller intervals than the first ones.

It is possible to identify two main styles that should be roughly followed: the *close position* and the *open position* (see the figure 10.3). When a triad is written in close position, no other triad's notes can be inserted between two adjacent parts. The close position is related to a brilliant and clear sound; if a spreader displacement is used, the sound becomes soft and mellow.

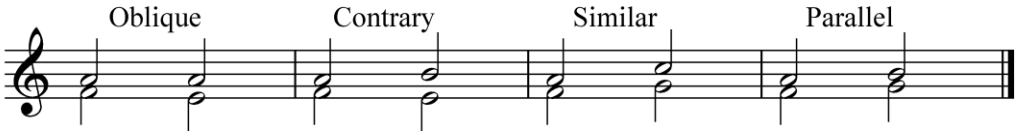


[**Figure 10.3**: Close position and open position.]

Since chords arise from the superposing of different and autonomous melodic layers, a very general principle of harmony is to maintain the **relative independence of the parts**. That is, the voices should differentiate their relative motions. Not all the voices should move upward or downward at the same time: at least one part has to differentiate its proceeding toward the next chord, that is, if three of four parts go up, the remaining part should go down, or be stationary, and vice-versa. To be specific, it is possible to distinguish four types of collective motion: **contrary motion**, **oblique motion**, **similar motion** and **parallel motion** (see the figure 10.4). The autonomy of the parts is obtained by avoiding a frequent use of parallel motion; in particular, contrary motion of the upper voices to the bass usually produces good results.

To be specific, whereas the parallel motion between two voices can be successfully adopted if the distance between the two voices is a third or a sixth, the parallel motion between the other consonant intervals, the octave, the fifth and also (to a lesser extent) the fourth, is generally to be avoided unless a big amount of instruments/voices are present (e.g., a whole orchestra). In the case

of the octaves and the fifths, the parallel movement is respectively called **parallel octaves** and **parallel fifths**.

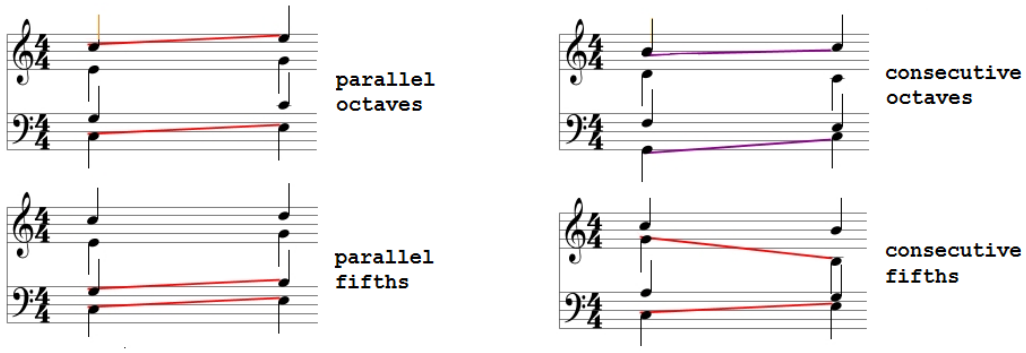


[**Figure 10.4**: Examples of contrary, oblique, similar, and parallel motion.]

The case of parallel octaves and fifths requires carefulness because of the following reason: parallel melodies separated by an octave interval always share exactly the same harmonics, and also the fifth interval involves a considerable amount of common harmonics. Consequently, in both the situations, the two parallel parts continuously sustain each other. Their parallel motion strengthen their melody. But, in the four-part writing, this parallelism compromises the homogeneity of the whole sound, and produces harshness. Observe that, if the harmonics are viewed as notes that are missing from the score, the parallel motion in octaves and fifths corresponds to the introduction of a much greater amount of parallel melodies. In fact, the quantity of harmonics that the notes of the octaves and the fifths intervals have in common is exceptionally bigger than that of any other interval.

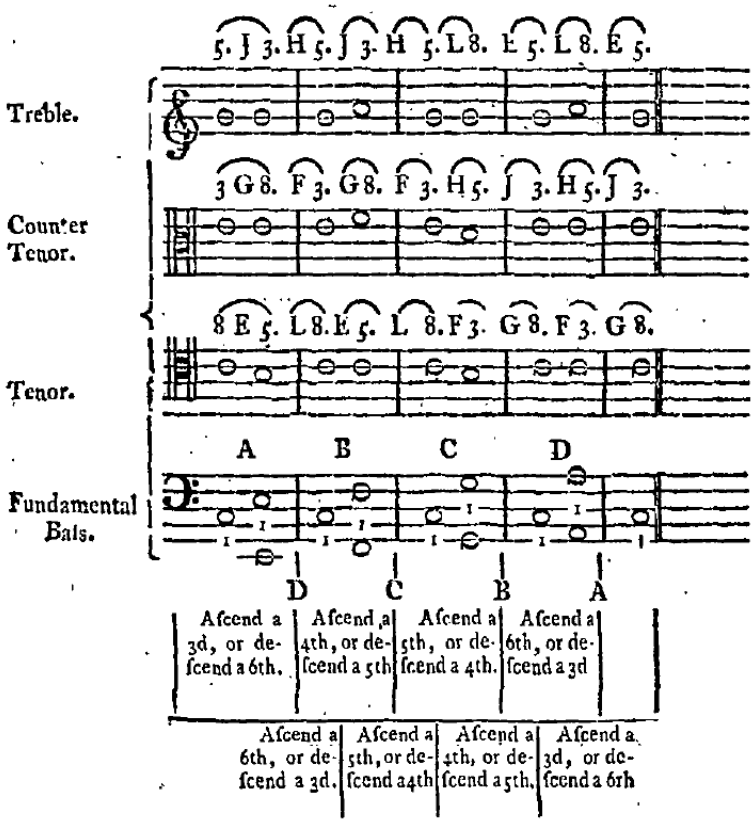
Thus, whenever two parts proceed with parallel motion, they should share a smaller amount of harmonics than fifths, octaves (and fourths), but they still have to form consonant intervals: that is, what remains are the thirds and the sixths. Parallel thirds and sixth are regularly used in the music of the common practice. In the case of big ensembles, on the other hand, the need of doubling the parts regularly leads to parallel motion in octaves: many elements will have to perform the same part, which usually must be pitched at different octaves depending on the instruments' ranges. Therefore, in this context, parallel octaves are largely used.

Whenever consecutive fifths or octaves are reached with non-parallel motion, they are called **consecutive fifths** and **consecutive octaves** (see figure 10.5). They are much less noticeable than the case of the parallel ones, and their presence is so frequent in the literature that any attempt to avoid them would be useless under every reasonable point of view.



[**Figure 10.5**: Parallel octaves, parallel fifths, consecutive octaves, consecutive fifths.]

The following figures give two examples, taken from Rameau's *Traitè de L'armonie*, where the above guidelines apply. Figure 10.6 shows the connections I-III, I-IV, I-V and I-VI.



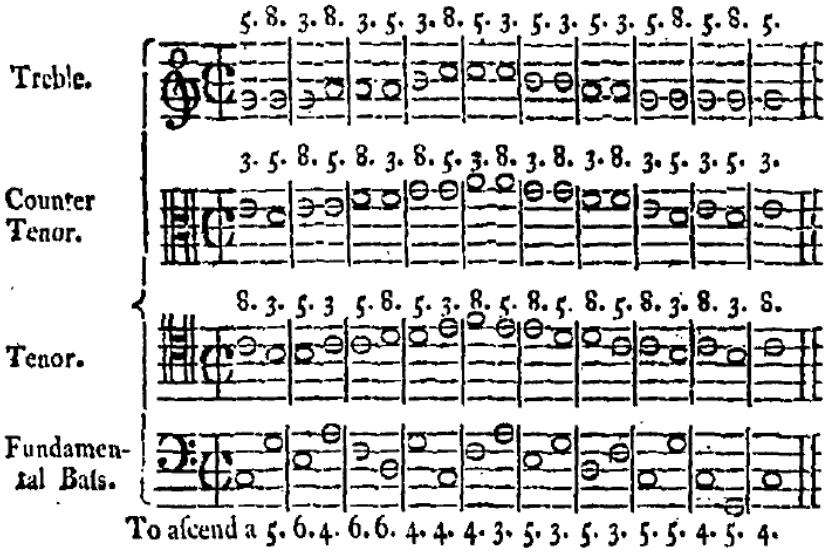
[**Figure 10.6**: Jean-Philippe Rameau, *Traitè de L'armonie* (1722). The inner parts use C clefs. The numbers above each note indicate whenever the note is the third of the triad (3), the fifth (5) or the root's octave (8).]

Figure 10.7 shows a basic musical phrase, where the lack of VII allows to avoid dissonances.

As it was anticipated in the previous chapter, in the case of chords including a dissonant interval, one of its notes should appear in the previous chord too - where it forms consonant intervals with the other notes (i.e. it is "prepared"). Then it remains held in the same part, so that when passing to the second chord it is "hidden" by the manifestation of the new other notes. That is, the prepared note is a common tone between the consonant and the dissonant chord. This practice was introduced also because it helps the singers: the dissonant intervals are usually difficult to sing at sight, so that putting the problematic note in the chord before makes possible to sing it when it's easy to intone.

The procedure for introducing dissonances can be iterated through a progression of dissonant chords (that is, the chord where the note is prepared can be dissonant too). When resolving the dissonance into a consonant triad, then, the note that was prepared usually moves one degree downward, and the bass (the root) often leaps to the fourth above (the fifth below). Otherwise, the dissonant note can ascend (e.g. when the bass is tied over to the next chord), it can be delayed

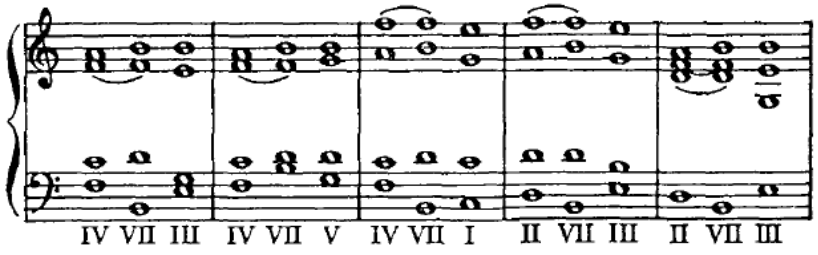
(when one or two chords intervene between the dissonant interval and its resolution), or it can leap if another dissonance follows.



[**Figure 10.7**: Jean-Philippe Rameau, *Traitè de L'armonie* (1722). The numbers on the bottom indicate the intervals that are taken by the bass's motion. Rameau always indicates them as ascending (e.g., if the bass proceeds a third downward, it is considered a leap of a sixth, and therefore it is indicated with the number 6).]

Both the preparation and resolution of dissonances should be performed by the same part, which begins and concludes into consonant intervals, and produces a dissonance only when it doesn't move. Moreover, doubling a dissonant note has to be avoided (also) because both the dissonant note and its octave-replication should resolve the same way, creating parallel octaves.

In the figure 10.8 examples of the regular treatment of the diminished triad's dissonance are shown.



[**Figure 10.8**: Examples of treating the tritone dissonant interval when appearing in the diminished triad on the seventh degree of the major scale. Observe the F generating the dissonance: it is already contained in the previous chord (VI or II) and remains tied when passing to VII. When escaping from VII, then, F descends to E.]

During the common practice period, the above method for leading the parts through the dissonances has been progressively weakened; since the ending of the sixteenth century, various exceptions have arisen, and the treatment of dissonance has become more freer. If a dissonant interval is introduced without preparation, then the two notes should enter by contrary motion, making the smallest possible movement (preferably one degree). However, to avoid the preparation or the resolution of the dissonances requires the awareness of the resulting effects: by properly preparing and resolving the dissonances, smooth and solid chord connections will certainly arise. The ability of writing fluent harmonies, then, can be exploited for to obtain much more complex artistic products, where the missing of the preparation and/or the resolution of the dissonances is the result of a conscious act. In other words, before taking part to a motor-bike competition, one should be able to ride a bicycle.

11-Voice-leading in minor tonality

In chapter 7 the harmonic and the melodic minor scales have been introduced. In A minor key, the harmonic minor scale is A, B, C, D, E, F, G#, and the melodic minor scale is A, B, C, D, E, F#, G#. These scales find their origin in the typical practice, during the modal era, of altering either the seventh or both the seventh and the sixth degrees of the minor-like modes, especially the Dorian and the Aeolian.

The chromatic raising of the seventh degree (G) of the A natural minor scale (the Aeolian mode) provides the leading-tone (G#) to the A minor tonality. As it will be discussed in chapter 15, the major scale's seventh degree is called leading-tone because it has the tendency to lead the melody to the above tonic (which is a semitone far from it). Thus, the chromatic change in the minor's seventh degree occurs only when the melody goes to the tonic. When descending from the tonic, on the contrary, the seventh degree is usually left unaltered (especially when proceeding by degrees, i.e., A, G, F, E, …). This applies mainly to the higher part (the leading melody), but also the bass and the inner parts often behave differently whenever they reach or they leave the tonic.

The chromatic raising of the sixth degree is a consequence of the change in the seventh degree: if only the seventh was raised, an augmented second (F-G#) would appear between the unaltered sixth and the altered seventh. Augmented and diminished intervals were forbidden in the modal counterpoint practice, and this tradition is partially survived in the tonal era, also because the augmented second is difficult to intone when singing at sight. Moreover, the F-G# interval produces a particular "oriental" flavor: it is included in the harmonic minor scale, and in fact such scale is the basis of many non-Western music.

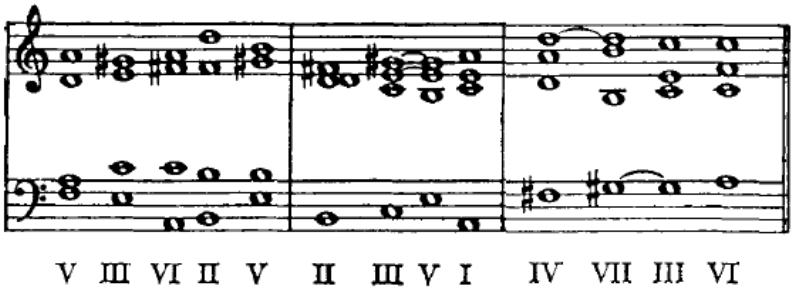
As a consequence of the chromatic alterations that characterize the ascending motions toward the tonic, the melodic minor scale is also called *ascending minor scale*, and the natural minor scale is also called *descending minor scale*. Such ascending/descending different behaviors lead to the following guidelines:

- If the melody takes G#, then it should ascend to A.
- If the melody takes G, then it often descends to F.
- If the melody takes F#, then G# should follow.
- If the melody takes F, then it often descends to E.

In figure 11.1 some examples are shown.

The above guidelines, however, may be in opposition with the procedure for the preparation/resolution of the dissonant intervals. E.g., the dissonance C-G# lying into the augmented triad C-E-G# (III of the harmonic and melodic minor scales) cannot be subjected to the standard resolution of the dissonance because G# leads to the tonic, i.e. a half-step upward: that is, G# resolves by moving to A instead of descending.

The change in the seventh degree of the natural minor scale can be regarded as a general procedure that allows to embed chromatically raised notes into a piece of tonal music: the chromatically raised note should be treated like it was the seventh degree of a harmonic or melodic minor scale, and thus it should move a semitone upward, as it went to a tonic. Similarly, a chromatic lowered note can be thought as the sixth degree of the natural minor scale, resolving down a half-step.



[**Figure 11.1**: Examples of the use of chromatic alterations of the sixth and seventh degrees in minor.]

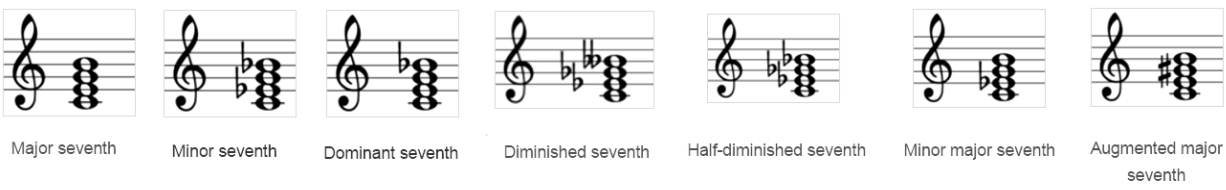
Moreover, when the note that gets the accidental is also included, without the accidental, in previous chord, then the same part should move from the unaltered to the altered note, especially if the root of the two consecutive chords is the same. The presence of the unaltered note and the altered note in different parts is called **false relation**, and should be avoided.

12-Seventh chords

Three seventh intervals are present in the major and minor scales: the minor seventh (e.g., the interval A-G in the A natural minor scale), the major seventh (e.g., the interval C-B in the C major scale), and the diminished seventh (e.g., the interval G#-F in the A harmonic minor scale). The augmented seventh would be e.g. C-B#, which has the same amplitude of the octave, and it is seldom used.

Among the chords of four notes, the seventh chords constitute a very important subset. They are obtained by placing another third above the two that form the triads, so that a seventh interval arises between the triads' root and the additional note. Therefore, they contain the root, the (major or minor) third, the (augmented or diminished) fifth and the (major, minor or diminished) seventh. To be specific:

- If a major third is placed upon a major triad, a **major seventh chord** is produced. E.g., C-E-G-B. It comprehends one dissonant interval, the major seventh C-B.
- If a minor third is placed upon a major triad, a **dominant seventh chord** is produced. E.g., C-E-G-Bb. It comprehends two dissonant intervals: the minor seventh C-Bb and the diminished fifth E-Bb.
- If a major third is placed upon a minor triad, a **minor-major seventh chord** is produced. E.g., C-Eb-G-B. It comprehends two dissonant intervals: the major seventh C-B and the augmented fifth Eb-B.
- If a minor third is placed upon a minor triad, a **minor seventh chord** is produced. E.g., C-Eb-G-Bb. It comprehends one dissonant interval, the minor seventh C-Bb.
- If a major third is placed upon a diminished triad, a **half-diminished seventh chord** is produced. E.g., C-Eb-Gb-Bb. It comprehends two dissonant intervals: the minor seventh C-Bb and the diminished fifth C-Gb.
- If a minor third is placed upon a diminished triad, a **diminished seventh chord** is produced. E.g., C-Eb-Gb-Bbb. It comprehends three dissonant intervals: the diminished seventh C-Bbb and two diminished fifths, C-Gb and Eb-Bbb.
- If a minor third is placed upon an augmented triad, an **augmented-major seventh chord** is produced. E.g., C-E-G#-B. It comprehends two dissonant intervals: the major seventh C-B and the augmented fifth C-G#.



[**Figure 12.1**: The 7 seventh chords: major, minor, dominant, diminished, half-diminished, minor-major, augmented-major.]

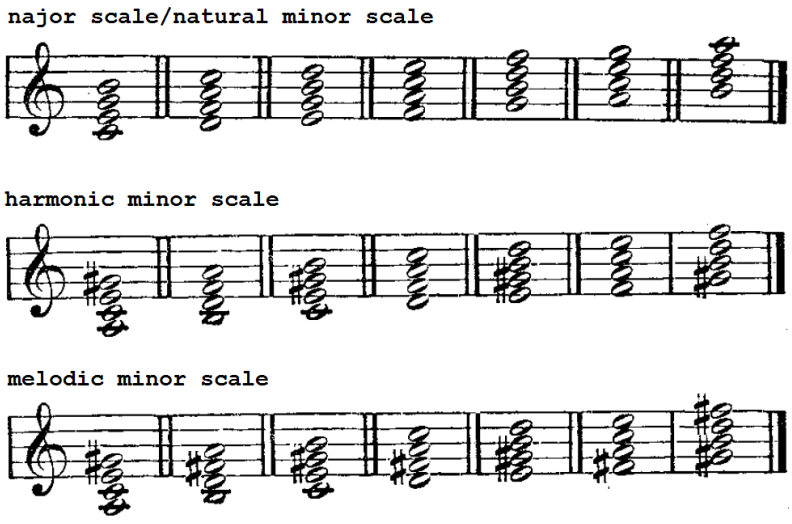
As the four types of triad are obtained by building a triad on the degrees of the major and minor scales, without using additional chromatic alterations (as discussed in chapter 6 and 7), also the seventh chords can be analogously produced without introducing further accidentals. To be specific, in the case of the C major scale, the seventh chords are the following:

- **C**-E-G-B, a major seventh chord.
- **D**-F-A-C, a minor seventh chord.
- **E**-G-B-D, a minor seventh chord.
- **F**-A-C-E, a major seventh chord.
- **G**-B-D-F, a dominant seventh chord.
- **A**-C-E-G, a minor seventh chord.
- **B**-D-F-A, a half-diminished seventh chord.

The seventh chords on the A natural minor scale's degrees, of course, coincide with the above ones.

In the case of the A harmonic minor scale, the presence of G# produces on the first degree a minor-major seventh chord (**A**-C-E-G#), on the third degree an augmented-major seventh chord (**C**-E-G#-B), on the fifth degree a dominant seventh chord (**E**-G#-B-D) and on the seventh degree a diminished seventh chord (**G#**-B-D-F). The others degrees involve the same seventh chords of the natural minor scale.

In the case of the A melodic minor scale, the presence of both F# and G# produces on the second degree a minor seventh chord (**B**-D-F#-A), on the fourth degree a dominant seventh chord (**D**-F#-A-C), on the sixth degree a half-diminished seventh chord (**F#**-A-C-E), and on the seventh degree another half-diminished seventh chord (**G#**-B-D-F#). The others degrees involve the same seventh chords of the harmonic minor scale.



[**Figure 12.2**: Seventh chords on the major and the minor scales.]

The scale's degree upon which the seventh chords are built is called *root*, analogously to the case of the triads. When the root is the bass, the chords are in root-position. Since the seventh chords have four notes, there are three possible inversions. If the third is placed in the bass, the chord is called a *six-five chord*, when the fifth is the bass it is called a *four-three chord*, and when the seventh is the bass, a *two chord*. As in the case of the triads, these names refer to intervals with the new bass.

Among the seventh chords, the dominant seventh chord is particularly important. From an acoustic point of view, it origins quite spontaneously. In fact, as we have seen, the major triad C-E-G involves the first two harmonics of C being different from the fundamental frequency's octaves: if the next new harmonic, the 7^{th} (Bb), is taken, then the dominant seventh chord C-E-G-Bb is produced. The minor seventh that is found between the 4^{th} harmonic (C) and the 7^{th} is called **harmonic seventh**.

The dominant seventh chord appears without the need of placing accidentals on the diatonic dominant degree: G-B-D-F. Observe that if the bass is omitted, then the diminished triad (VII) is obtained. Moreover, the half-diminished seventh chord on the diatonic seventh degree, B-D-F-A, can be interpreted as a dominant seventh chord where the bass is omitted and the 9^{th} harmonic (the next new harmonic of G after the 7^{th}, that is, A) have been added.

Observe that a major **sixth chord**, i.e. a major triad plus the sixth degree of the major scale, is a seventh chord: e.g., the triad C-E-G-A is one of the inversions of A-C-E-G, which is a minor seventh chord. Moreover, the sixth chord C-Eb-G-A is one of the inversions of A-C-Eb-G, which is a half-diminished seventh.

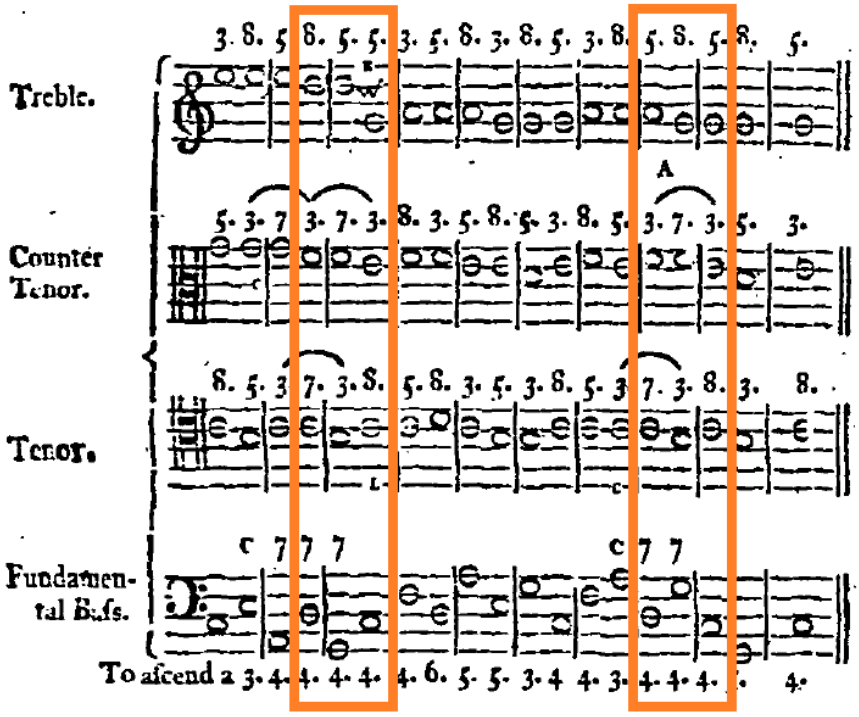
The seventh chords usually resolve their dissonant seventh interval by letting the seventh to move one step downward, to become the third or the fifth of the next chord. It is also frequent to keep the seventh sustained while the bass ascends. In the figure 12.4, taken from Rameau's treatise, the most typical resolution of the dominant seventh chord is shown. Figure 12.5 and figure 12.6 give some examples of treatment of the seventh chords' dissonances. In figure 12.7, some examples in minor, and in the figure 12.8 two little phrases in minor.

Whenever a chromatically raised note is the seventh of a seventh chord, the laws for the voice-leading are in conflict with the standard treatment of the dissonances, as in the case of the augmented triad. Namely, considering A minor, the chords G-B-D-F#, A-C-E-G# and G#-B-D-F# involve G# and F# as the seventh of the chord: as dissonances, they would descend, but as chromatically raised notes they should take an upward motion.

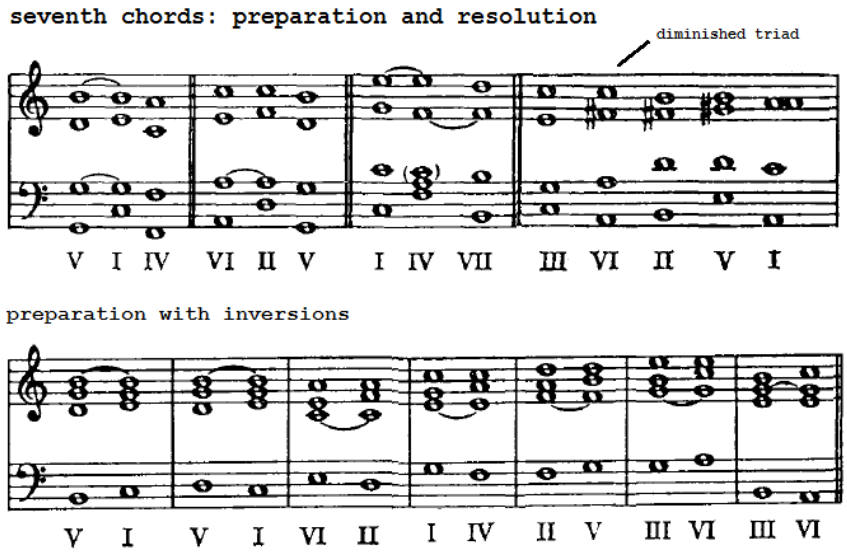
Monteverdi is one of the firsts that freely introduced unprepared dominant seventh chords in its works. Later, Bach did the same with the other seventh chords.

The earliest laws of harmony would also forbid the resolution of the seventh interval into the octave: i.e., a seventh whose lowest note descends a step while the upper note is sustained, or vice-

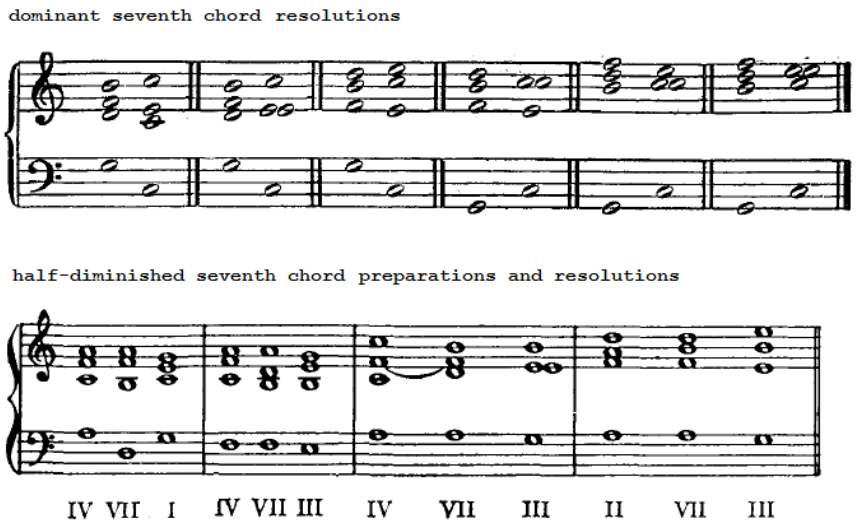
versa, the upper note ascends while the lowest note is sustained. As such exceptions began to appear in the masterpieces, they became accepted by the theory.



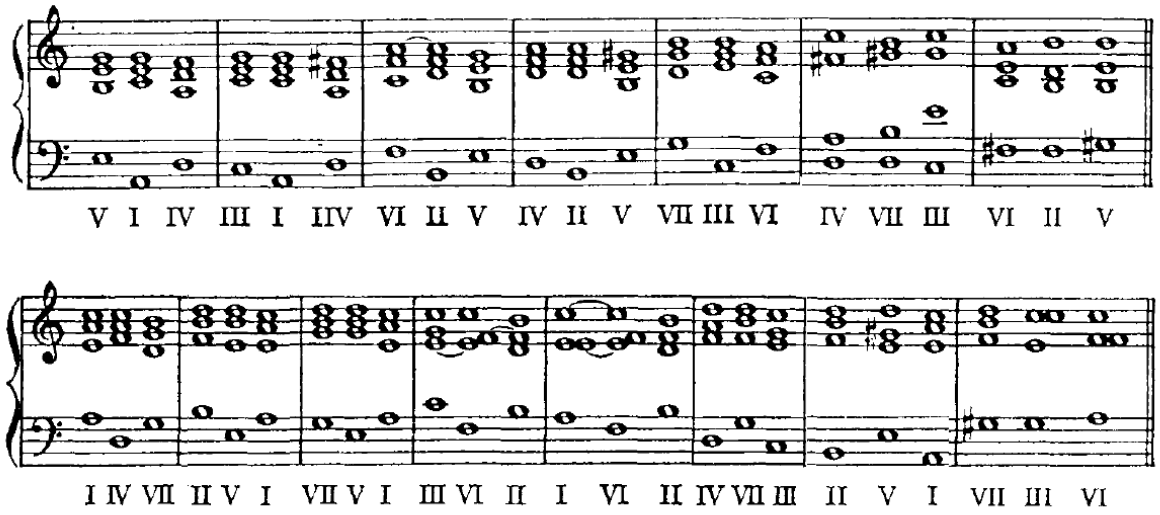
[**Figure 12.4**: Rameau's example from *Traitè de L'armonie* (1722). The rectangles mark the customary progression (VI)-II-V-I, where the dominant seventh chord lies on the dominant degree. The note F (which forms two dissonant intervals, B-F and G-F) is prepared in II and resolved in I by descending to E. In the second rectangle, II is again used to include F as a consonant interval, but this time II is a seventh chord too, and its seventh (C) is prepared in VI.]



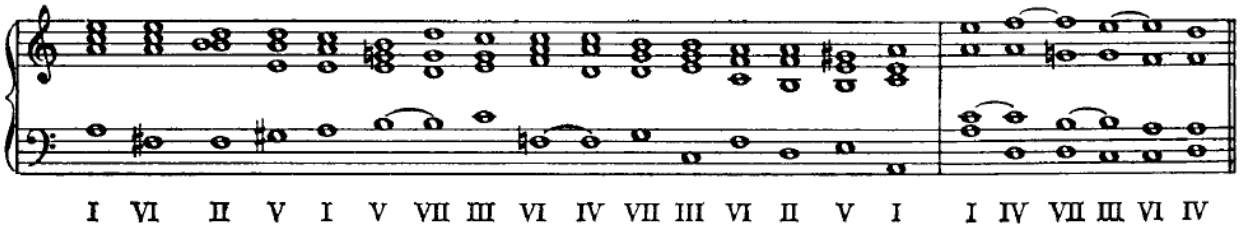
[**Figure 12.5**: Preparation and resolution of the seventh chords with chords in root position, and preparation with inversions.]



[**Figure 12.6**: Examples of dissonance preparation and resolution for the cases of the dominant seventh chord and the half-diminished seventh chord.]



[**Figure 12.7**: Arnold Schoenberg, *Theory of Harmony*. Preparation and resolution of seventh chords in A minor tonality.]



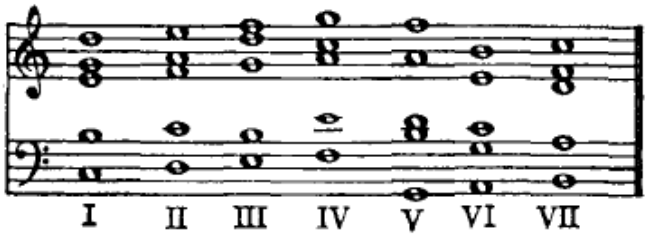
[**Figure 12.8**: Arnold Schoenberg, *Theory of Harmony*. Phrases in minor tonality.]

13-Ninth chords

The ninth chord is made up by placing another third above a seventh chord, so that a major or minor ninth interval with the bass is produced. As in the case of the seventh chord, taking the major scale, the following ninth chords spontaneously appear (see also figure 13.1):

- **C**-E-G-B-D
- **D**-F-A-C-E
- **E**-G-B-D-F
- **F**-A-C-E-G
- **G**-B-D-F-A
- **A**-C-E-G-B
- **B**-D-F-A-C

They comprehend the third, the fifth, the seventh and the ninth degrees above the bass (the root). Those on the third and seventh degrees involve a minor ninth interval, the others a major ninth interval. The major ninth coincides with the 9^{th} harmonic of the bass: therefore, the ninth chord on the dominant degree, G-B-D-F-A (where the first four notes form a dominant seventh chord), comprehends all the first 9 harmonics of its bass.



[**Figure 13.1**: Ninth chords on the degrees of the diatonic scale.]

One of the characteristic of these chords is that their inversions are rarely used, because often sound really bad. However, there are numbers of examples in the twentieth century's literature.

All the ninth chords on the major and minor scales comprehend another seventh chord, which is obtained by omitting the bass. E.g., taking the ninth chords on the diatonic scale (listed above), if the root is left out from the chords on the first, fourth and seventh degrees, a minor seventh chord is obtained (respectively E-G-B-D, A-C-E-G and D-F-A-C); on the second and sixth degrees a major seventh chord (F-A-C-E and C-E-G-B), on the third a dominant seventh chord (G-B-D-F), and on the fifth a half-diminished seventh chord (B-D-F-A).

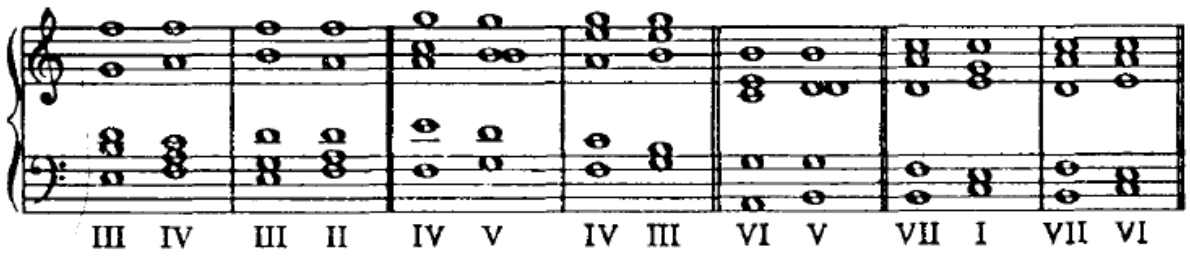
If the A harmonic minor scale is considered, the following new chords arise from the presence of G#:

- **A**-C-E-G#-B
- **C**-E-G#-B-D
- **E**-G#-B-D-F
- **F**-A-C-E-G#
- **G#**-B-D-F-A

and if the melodic minor scale is considered, the following new chords appear:

- **B**-D-F#-A-C
- **D**-F#-A-C-E
- **E**-G#-B-D-F#
- **F#**-A-C-E-G#
- **G#**-B-D-F#-A

The ninth intervals' dissonance is subjected to the same treatment of the seventh intervals. Thus, the ninth, the seventh and the diminished fifth (if present) all move one step downward (see figure 13.2). These dissonances can also be sustained while the bass ascends.



[**Figure 13.2**: Examples of ninth chord's resolutions.]

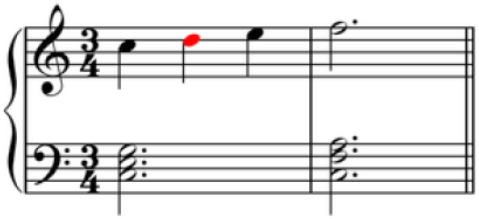
The omission of both the seventh and the third in the chords with the major ninth produces the second inversion of the **perfect-fourth chord**, a chord made up by adding two perfect fourth. E.g., the omission of E and B in the ninth chord C-E-G-B-D produces the second inversion of D-G-C. This second inversion, C-G-D, where D is the highest note, is a quite pleasant chord: in fact, G is the 3rd harmonics of C, and D is the 3rd harmonics G. The lack of the third allows this chord to be interpreted either as a major or a minor chord, and, although the ninth is technically a dissonance, it often doesn't need preparation. This inversion is very frequently used in the current popular music, where C and G are often doubled.

The second inversion of the perfect-fourth chord suggests that the perception of the major second interval considerably changes when an octave is added to it (to obtain a major ninth), while, in general, this doesn't apply to the other intervals. In fact, the 9th harmonic lies much higher (more than three octaves) than its fundamental: consequently, the major ninth resembles the displacement of the harmonics much more closely than the major second. Observe that, for the same reason, the presence of small intervals above the bass generally sounds bad, as noted in chapter 10.

Moreover, the first inversion of the perfect-fourth chord D-G-C, i.e. G-C-D, includes a fourth (G-C) above the bass (which sounds dissonant) and a major second C-D: both these intervals are usually resolved with a descending motion of the fourth C to the third of the major triad G-B-D (while the other two notes remain stationary). This is one of the most adopted dissonance resolutions in Western music.

14-Embellishing notes

Since the early times of the development of harmony, the introduction of *passing notes* between two consonant chords has been an ordinary practice. When a part makes a leap e.g. of a third or a fourth interval, these intervening notes, usually positioned on the weak beats (or weak part of the beats), involve the remaining diatonic degrees between the interval's lower and upper notes. During this (ascending or a descending) melodic motion, the passing notes form dissonant intervals with the other (stationary) parts.



[**Figure 14.1**: Example of passing notes between consonant chords. The red note forms a dissonant interval with both the root and the third of I.]

The degree to which an interval is perceived as dissonant also depends on how well its notes are integrated into the horizontal dimension of the music. From this point of view, the historical origin of the use of dissonant chords, in particular the seventh chords and the ninth chords, is frequently recognized in the incorporation of the passing notes into the chords that they melodically connect.

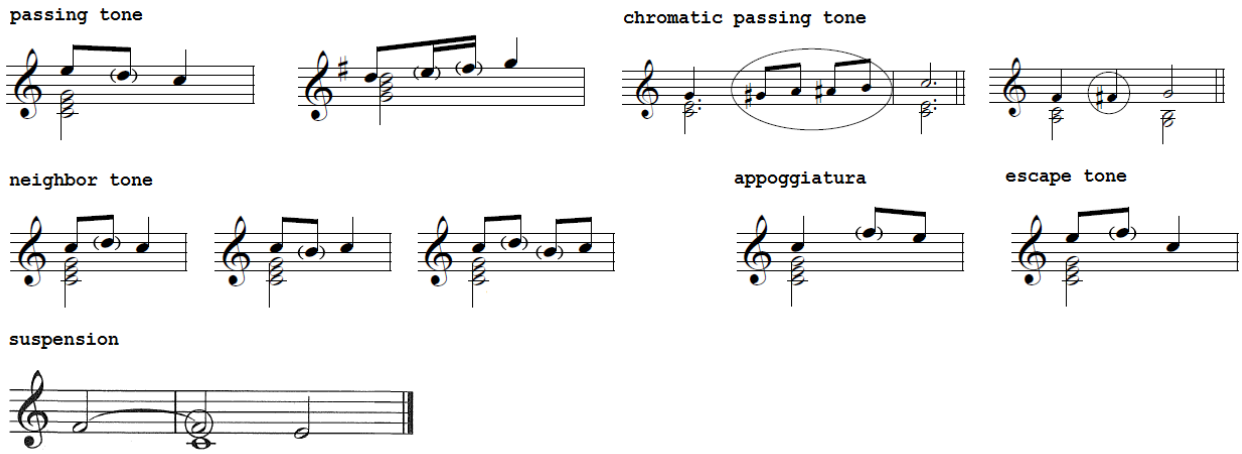
In fact, although the passing notes have an "ornamental" function, their introduction is a valid method to produce harmony: when some notes are listened one after the other, the brain spontaneously evaluates the relationships between them, and perceives the chord that they would create if they were sounded simultaneously. In particular, when a chord is produced by a sequence of consecutively sounded notes, it is called **arpeggio**.

The embellishment notes are usually distributed in alternating voices, and provide a useful device for increasing the dynamics and the fluentness of the voice-leading. They follow a pattern of consonance-dissonance-consonance, which creates a brief moment of tension followed by relaxation, and the degree of dissonance is increased if it falls on a metrically strong position. Although some of the ornamental notes may themselves be members of a chord, they have a secondary importance, and could be discarded without changing the harmonic character of a musical passage.

Among the most common embellishment notes, also called **non-chord tones**, **non-harmonic tones** or **figuration tones**, the following ones have widely accepted names (see figure 14.2):

- **Passing tone**: a passing note that fills a gap between two notes a third apart. Two adjacent passing tones can also be used to connect notes a fourth apart.

- **Chromatic passing tone**: a passing note the fills a gap between two notes a tone apart, or, in the case of larger intervals, adjacent passing tones that form a fragment of the chromatic scale.
- **Neighbor tone**: a passing tone that occurs between two instances of the same note. It can also consist of two notes, one a step above and the other a step below the chords' common note, making a *neighbor figure*.
- **Appoggiatura**: a passing note that is approached by leap and proceeds by step to the second chord. It is also called **incomplete neighbor tone**.
- **Escape tone** (or **echappée**): the opposite motion to that of the appoggiatura; that is, a passing note which is approached by step and proceeds by leap to the second chord. Also the escape tone is sometimes called **incomplete neighbor tone**.
- **Suspension**: a particular case of resolution of the dissonance. It arises whenever the resolving chord corresponds to the dissonant chord, except for the dissonant note, that resolves downward. That is, all the other notes remain sustained, and the dissonant note is the only one that moves.
- **Retardation**: an upward-resolving suspension.



[**Figure 14.2**: Non-chord tones.]

15-Tendency tones and cadences

Three major triads can be obtained from the seven degrees of the major scale: the tonic (I), the subdominant (IV) and the dominant (V). As noted in chapter 6, these chords are the most significant elements among the major scale's triads. Although all the seven triads spontaneously converge to the tonic triad, the dominant and the subdominant chords provide the two most incisive connections to I: V-I and IV-I. In particular, the greater predisposition of the dominant degree to move toward the tonic confers particular vigor to V-I, which is stronger than IV-I.

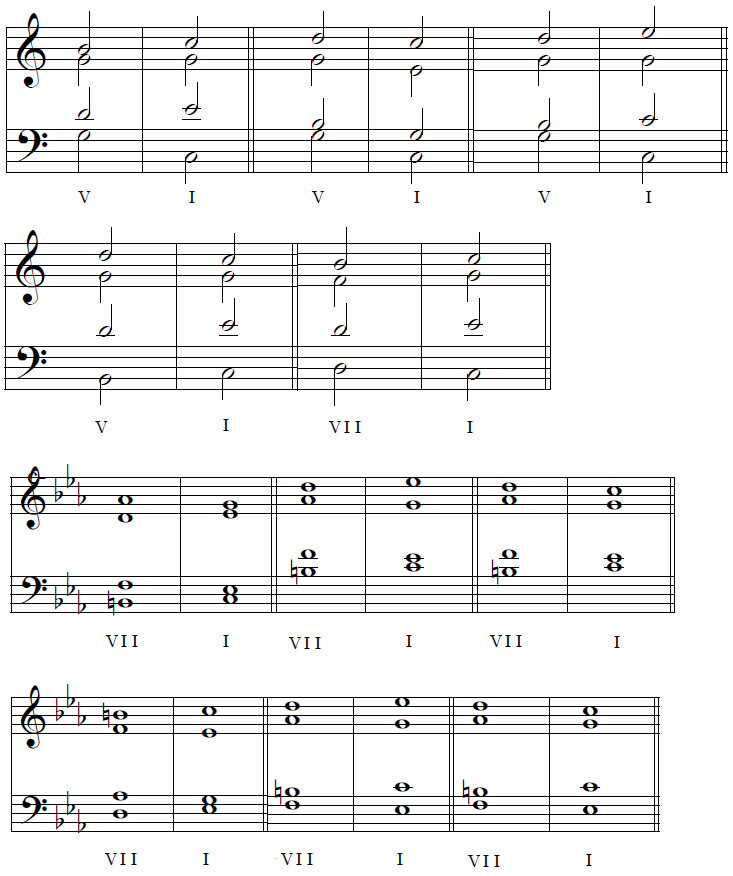
There is a fundamental element that makes both the progression V-I and IV-I particularly stable and solid. Consider the major tonality: the major scale comprehends two half-steps, the first lies between the third and fourth degrees, the second between the seventh degree (the leading-tone) and the tonic. When connecting V-I and IV-I, these semitone-intervals assume a melodic function: the leading-tone (the third of V) spontaneously ascends to the tonic (the root of I), and the fourth degree (the root of IV) spontaneously descends to the third degree (the third of I). E.g., in C major, when connecting V (G-B-D) to I (C-E-G), the leading-tone B goes to C; when connecting IV (F-A-C) to I, F goes to E. The presence of the half-steps is crucial for making natural these motions.

The notes that behave like the major scale's seventh and fourth degrees are called **tendency tone**s. A tendency tone is a note lying a semitone away from a second note, so that it spontaneously needs to be followed by this second note: this movement is also called "resolution" of the tendency tone, since it is somewhat similar to the resolution of a dissonance. The resolution of the tendency tones provides the melodic counterpart to the progressions V-I and IV-I.

In the case of minor tonalities, the leading-tone (in A minor, G#) is supplied by the melodic and harmonic minor scales (this is precisely the historical reason of their introduction). Thanks to G#, the A minor's dominant triad becomes major (E-G#-B), so that, in the progression V-I, G# leads to A. The other tendency tone of A minor is F, which resolves going to the fifth of the tonic triad. The semitone-interval B-C, when adopting the A minor scale(s), doesn't have a strong melodic weight: the motion from B to C (the third of A minor's tonic triad) produces a weak sustainment to V-I.

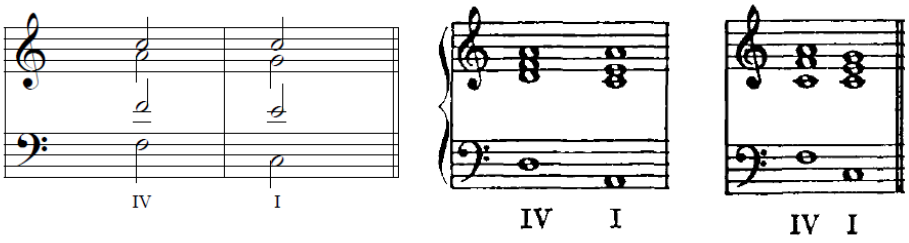
Both in major and minor keys, the progression V-I from the (major) dominant triad to the tonic triad is called **authentic cadence** or **perfect cadence** (shown in figure 15.1). This connection is extremely solid, and the motion to the tonic triad is associated to a strong sense of conclusion, especially when the triads are in root position and the leading-tone is assigned to the highest part.

When designing an authentic cadence, the minor seventh is usually added to V, producing a dominant seventh chord (in C major, G-B-D-F; in A minor, E-G#-B-D). Thanks to this additional dissonant element, the cadence is reinforced by the resolution of the seventh interval, and, in major key, also by the resolution of the other tendency tone (the seventh itself). Furthermore, by omitting the root of the dominant seventh chord on the fifth degree, the diminished triad on the leading-tone is obtained. Consequently, V-I can be replaced by VII-I. Also, a minor seventh is usually placed upon VII, so that it becomes a half-diminished seventh chord.



[**Figure 15.1**: Authentic cadences.]

The transition from the subdominant triad to the tonic triad (IV-I) is called **plagal cadence** (shown in figure 15.2). It is a more gentle and solemn progression than the authentic cadence. Christian church hymns ends with the word "Amen", which is almost always sustained by the plagal cadence. The tendency tone being involved in this cadence is the diatonic's fourth degree. As noted above, in the major keys it resolves by moving to the third of the tonic triad, whereas in the case of minor keys it resolves to the fifth. Observe that, in minor tonalities, IV is not affected by the accidental on the six degree that characterizes the melodic minor scale, and then it remains a minor triad.

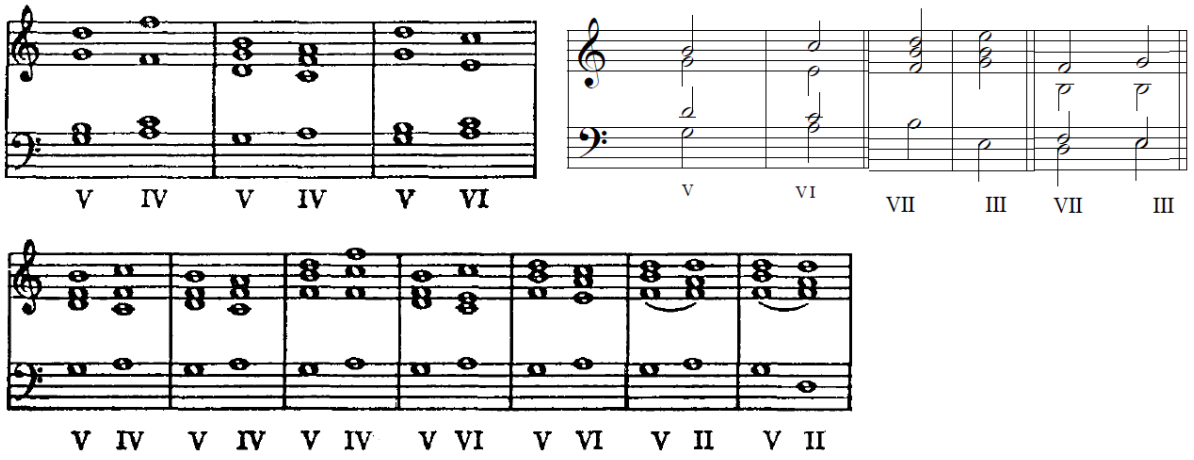


[**Figure 15.2**: Plagal cadence.]

In the music of the common practice period, the subdominant triad is frequently placed before V in the authentic cadence, producing IV-V-I. This variant is one of the strongest chord progressions in tonal music. In minor keys, particular carefulness is needed when connecting the subdominant to

the (major) dominant triad: an augmented second can be produced between the (unraised) sixth degree (the third of IV) and the chromatically raised seventh degree (the third of V).

A large amount of common practice music ends through the authentic cadence. Whenever the chord following V (or VII) is not I - which is the chord the listener was expecting - the resulting cadence is called **deceptive cadence** or **irregular resolution** (shown in figure 15.3). Typical deceptive cadences are V-VI and V-IV (or VII-VI and VII-IV).



[**Figure 15.3**: Deceptive cadences.]

Every chromatically raised or lowered note behaves like a tendency tone, both in consonant and dissonant chords. As will be discussed in the next chapter, chromatically raised notes should move one semitone upward (that is, they behaves like a leading-tone), and chromatically lowered notes should move one semitone downward (they behaves like the diatonic's fourth degree).

Both the chromatic alteration of a note and the resolution of the resulting tendency tone should take place in the same voice. That is, the unaltered note should connect directly with the altered note, and the last with its resolution. Moreover, the tendency tones shouldn't be doubled over different octaves, because their simultaneous resolutions would produce parallel octaves.

Observe that the ascending or descending motion that characterizes the nature of the chromatically altered notes is comprehended in the guidelines for the voice-leading in minor keys (discussed in chapter 11): every chromatically raised note should be treated as it was the seventh degree of the (ascending) melodic minor scale (or the sixth, if the next note is chromatically altered too), and every chromatically lowered note should be treated as it was the sixth degree of the (descending) natural minor scale.

16-Modulations

In tonal music, a modulation is the introduction of a new tonality. That is, a new diatonic scale is established by chromatically raising or lowering the notes of the old one, and the listener perceives the new tonic and the corresponding (major or minor) tonic triad as the most stable and conclusive sound.

A modulation doesn't have to necessary be a complete transition to a new key. It can leave the old tonality alive; it can be a transient and momentary event, involving only few chords. Two tonalities can cohabit together: the new tonic may not prevail over the former, some foreign notes may appear for a short period and then disappear, allowing the original tonic to spread his influence again.

Some authors, however, refer to the modulation only as a complete change of tonality: according to the principle of *monotonality*, any movement that involves new notes and brings the listener away from the tonic wouldn't be a modulation unless a new tonality is fully established. The departures from the tonal center, in fact, can be analyzed from an historical point of view, that is, in terms of the diatonic modes (listed in chapter 1) where the chromatically altered notes and related triads come from. During the modal era, the modes were frequently modified to obtain the tendency tones: the minor seventh was chromatically raised in the scales where the leading-tone is absent (particularly the Dorian, the Aeolian and the Mixolydian), and also the fourth degree of the Lydian was often chromatically lowered, so that the corresponding tendency tone was obtained.

Depending on the philosophy being adopted, therefore, the temporary digressions from the tonic that arise from the introduction of altered notes can be regarded as either a partial modulation to a different key or the adoption of a diatonic mode, under the influence of the same tonic.

When a new tonality is fully adopted, anyway, a considerable change takes place in the music, and everything happens after the change is emphasized by the new colors coming into sight. It is very common to use modulations for introducing new themes (musical phrases), so that e.g. the reprise of a theme is remarked by the return of its own tonality, or the same theme can have both a major-key version and a minor-key version.

The transition from one tonality to another usually can't happen all at once. An unexpected change of tonality must have strong artistic purposes: whenever the notes which are typical of the new key don't be introduced through an appropriate sequence of chords, the modulation is weak and undecided. The listener has to lose the reference sounds of the tonality that he is leaving, the music has to take the distances from the old tonic and it should enter in a phase of transition, where notes of both the old and the new tonalities are largely involved. In other words, the attraction of the old tonal center should progressively become weaker, providing the conditions for a new tonal center to be established.

As a consequence, the adoption of a new tonality becomes harder as the number of notes that have to change increases, that is, as the distance in the circle of the fifths (see chapter 6) between the old and the new keys augments. At the end of the transition, a strong chord progression should establish the new tonic. As discussed in the previous chapter, the authentic and plagal cadences provide solid connections to the tonic triad.

Both the cadences are strengthened by the resolution of the tendency tones (the seventh and the fourth degrees of the diatonic scale): the leading-tone moves to the tonic (in minor tonality the leading-tone is chromatically introduced), and the fourth degree moves either to the third (in major) or to the fifth (in minor) of the tonic triad. Since the leading-tone ascends, and the diatonic fourth degree descends, the leading-tone is also called *ascending tendency tone*, and the fourth degree is also called *descending tendency tone*.

Looking at the circle of the fifths, the tendency tones are the notes that change when going from a major tonality to either the next or the previous one in the circle. E.g., when going from C major to G major, a sharp on F is added: F is the descending tendency tone of C major, and the accidental produces the ascending tendency tone (F#) of G major. Going in the opposite direction, from C major to F major, a flat on B is added: B is the ascending tendency tone of C major, and the accidental produces the descending tendency tone (Bb) of F major.

Thus, in general, when going around the circle of the fifths in the direction where the number of sharps increases, the sharp that is added upon the fourth degree of a major tonality produces the seventh degree (the leading-tone) of the next major tonality. On the contrary, going in the direction where the number of flat increases, the flat that is added upon the seventh degree of a major tonality produces the fourth degree (the descending tendency tone) of the next major tonality.

The tendency tones drive the circular relationship. They lead from one tonality to another, being the notes that change in the transition from one key to adjacent ones. The standard way for passing to a new tonality, therefore, consists of the introduction (by adding accidentals or by cancelling those in the key signature) of its tendency tones, so that they can be exploited for reaching the new tonic triad through an authentic or plagal cadence, i.e. a chord progression sustained by a melodic motion. Not only V and IV of the target tonality can be used for the cadential connection with I: every chord containing the tendency tones can work in such a way. The introduction of a tendency tone in a chord, in fact, makes this chord to "play the role" of dominant or subdominant triad of a new tonality, so that a variant of the authentic or plagal cadence can be implemented.

The primary role of the tendency tones explains why the chromatically raised notes should move a half-step upward, and the chromatically lowered notes should move a half-step downward: in the first case, the raised note behaves like a leading-tone, so that it resolves by moving upward to the hypothetical tonic of a new tonality; in the second case, the lowered note behaves like the

descending tendency tone, so that it resolves by moving downward to the third (or fifth) of the hypothetical new tonic triad.

It is to be observed that the diatonic triads change their gender (i.e. they transform from major to minor, or vice-versa) whenever the tendency tones of the closest tonalities are introduced. To be specific, consider the C diatonic scale:

- If F, C, G and D are replaced with F#, C#, G# and D#, which are respectively the ascending tendency tones of G, D, A, and E major tonalities, then the minor triads II, VI, III, and the diminished triad VII, all become major triads. In particular, these new major triads are precisely the dominant triads of G, D, A, and E major tonalities (the keys where the tendency tones came from) .
- If B, E, and A are replaced with Bb, Eb, and Ab, which are respectively the descending tendency tones of F, Bb and Eb major tonalities, then the three major triads V, I and IV become minor triads. In particular, these new minor triads are precisely the subdominant triads of D, G and C minor tonalities, i.e., the parallel minor of F, Bb and Eb major tonalities (the keys where the tendency tones came from).

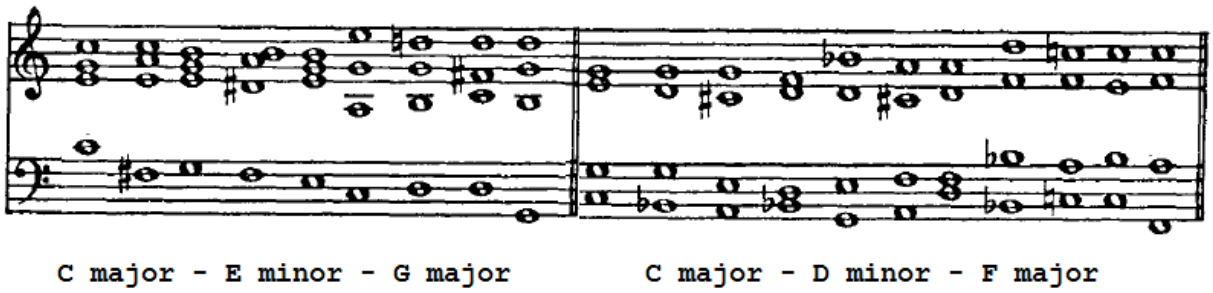
Thus, in other words, the introduction into the C diatonic scale of the leading-tones of G, D, A, and E major tonalities (which are, in the circle of the fifths, the closest to C major among the keys with the sharps) produces the dominant triads of such tonalities, and the introduction of the descending tendency tones of F, Bb and Eb major tonalities (the closest to C major among the keys with the flats) produces the subdominant triads of the parallel minor of such tonalities.

The (major) triads which are used as dominant triads of a new tonality, especially those that have become major triads by chromatic alteration, are called **secondary dominant**s. The secondary dominants allow to obtain for every degree a dominant chord to precede it. The chord that the secondary dominant spontaneously resolves to (through an authentic cadence) is called **tonicized chord** because the cadence makes it to be perceived as a tonic. Thus, the secondary dominants are involved in all the modulations where the new tonality's dominant triad is used for establishing the new tonic (in the common practice music, the large majority of the modulations).

Observe that the chromatic alteration that transforms the diatonic triads from minor to major, or vice-versa, from major to minor, introduces the altered note by means of consonant intervals (they are either a major or a minor third of a triad).

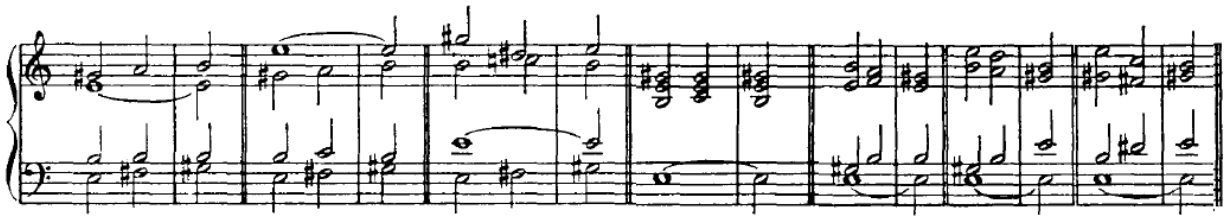
Since the tendency tones which are used for modulating belong to the tonality where the modulation brings to, and thus their meaning doesn't be clearly established at the moment of their entry, they can be introduced through **enharmonic change** or **enharmonic reinterpretation**, i.e. the rewriting of an altered note by using a different accidental (e.g., to write D# in place of Eb, or vice-versa). The enharmonic change alters the nature of the tendency tone, which transforms from ascending into descending, and vice-versa.

In the case of tonalities which lie far from each other in the circle of the fifth, the scarcity of common notes requires to find additional columns for sustaining the bridge between the old and the new tonic triads. In such cases, a third tonality can appear during the transition. This "passing tonality" is often called *intervening tonality*, and it should share some notes with both the initial and the final tonalities. It often happens that the intervening tonality is obtained by transforming a minor key into the major key of the same name (e.g. C major into C minor), or vice-versa, and then the latter is used for the modulation to the target tonality. The introduction of a third tonality can take place also when making simpler modulations. Since a diatonic scale is related to both a major and a minor tonality, then a third tonality is spontaneously available. E.g., when going from C major to G major, both A minor and E minor can be inserted in the middle.



[**Figure 16.1**: Arnold Schoenberg, *Theory of Harmony*. Examples of modulations.]

A frequently adopted method for taking the distances from the tonic consists of sustaining a component (usually the root) of either the dominant triad or a secondary dominant of the tonality that it's leaving, while the other voices proceed with a progression of chords that may also be dissonant with the sustained note. Whenever it is the bass, as it often happens, the sustained voice is called **pedal point**.



[**Figure 16.2**: Arnold Schoenberg, *Theory of Harmony*. Examples of pedal point.]

17-Diminished seventh chord and augmented triad

The diminished seventh chord and the augmented triad have some characteristics that make them special. Their construction, in fact, is very particular: the diminished seventh chord is made up by joining three minor thirds together; the augmented triad is made up by joining two major thirds. Consequently, the diminished seventh chord divides the octave in four equal parts (the sum of four minor thirds is equal to 12 semitones), and the augmented triad divides the octave in three equal parts (the sum of three major thirds is equal to 12 semitones as well).

E.g., consider the diminished seventh chord B-D-F-Ab. All its notes lie a minor third distant one from each other, and if another minor third was placed above Ab, Cb would be obtained: by enharmonic change (see the previous chapter), Cb can be written as another B. Similarly, by adding a major third on the top of the augmented triad C-E-G#, B# is obtained, which can be rewritten as another C.

As a result of these singular facts, the inversion of a diminished seventh chord is another diminished seventh chord, and the inversion of an augmented triad is another augmented triad. Therefore, by exploiting all the 12 semitones of the octave, it is possible to write only three different diminished seventh chords, and only four different augmented triads (shown in figure 17.1). To be specific, taking e.g. the first three degrees of the chromatic scale, C, C# and D, all the available diminished seventh chords are the following:

- C-Eb-Gb-Bbb
- C#-E-G-Bb
- D-F-Ab-Cb

All the other diminished seventh chords which are built on the remaining chromatic scale's degrees coincide with the inversions of one of the above three (by enharmonic changing some notes).

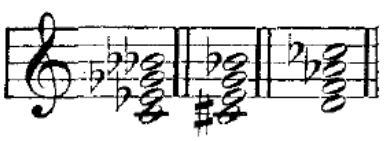
Similarly, taking the first four chromatic degrees, all the available augmented triads are:

- C-E-G#
- Db-F-A
- D-F#-A#
- Eb-G-B

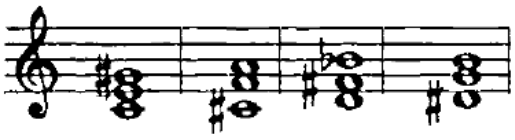
and those which are built on the remaining chromatic degrees coincide with the inversions of one of the above ones (by enharmonic changing some notes). Because of the symmetric nature of the augmented triad and the diminished seventh chord, the root (the bass) loses all its most important attributes. In fact, whenever the bass in changed, the amount of harmonics that the upper notes have in common with it remains the same.

Both the diminished seventh chord and the augmented triad naturally arise from the minor tonality: as shown previously, the diminished seventh chord can be obtained by building a seventh

chord on the seventh degree of the harmonic minor scale (in A minor, G#-B-D-F), and the augmented triad is obtained by placing a triad on the third degree of the harmonic or the melodic minor scales (in A minor, C-E-G#).



the 3 diminished seventh chords



the 4 augmented triads

[**Figure 17.1**: The three diminished seventh chords and the four augmented triads.]

The enharmonic reinterpretation of the notes of these two chords allows to consider them members of many tonalities. E.g., the diminished seventh chord G#-B-D-F can be written as:

- G#-B-D-F: it belongs to A minor.
- G#-B-D-E#: it belongs to F# minor.
- G#-B-C##-E#: it belongs to D# minor.
- G#-A##-C##-E#: it belongs to B# minor.
- Ab-B-D-F: it belongs to C minor.
- Ab-Cb-D-F: it belongs to Eb minor.
- Ab-Cb-Ebb-F: it belongs to Gb minor.
- Ab-Cb-Ebb-Gbb: it would belong to an hypothetical Bbb minor.

Those involving double-sharps or double-flats are rarely or never used.

Analogous considerations allow to see also the augmented triad from at least three different tonalities:

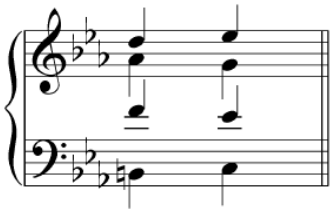
- C-E-G#: it belongs to A minor.
- B#-E-G#: it belongs to C# minor.
- C-E-Ab: it belongs to F minor.

Observe that the fifths of the above triads, G#, B# and E (which, in F minor, is written by placing a natural sign), all are chromatically raised tones (hence the name “augmented triad”).

Consider the diminished seventh chord. This chord is typically used for expressing passion, tension or other strong feelings. Since it can be enharmonically spelled in various ways, and thus it can be interpreted as an element of various tonalities, it can be connected with any other chord. Moreover, it can be introduced without preparing its dissonances. These characteristics provide to this chord the ability to bring distantly related chords, i.e. chords belonging to different tonalities, closer to each other. In other words, thanks to its indefinite character, the diminished seventh chord is frequently used for mitigating chord connections that otherwise would be forced (and

would sound harsh). It has the ability to turn away from the tonality, and, if used with moderation, it provides very good results.

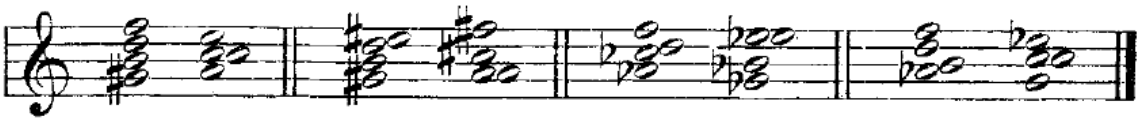
The diminished seventh chord can be interpreted as a minor ninth chord with the bass omitted. E.g., G#-B-D-F can be viewed as E-G#-B-D-F without E. Thus, if E is considered its bass, the most natural resolution would be on the tonic of A minor through the cadence V-I (as shown in figure 17.2, in the case of C minor). Observe that G#-B-D-F comprehends both the tendency tones of A minor, G# and F. However, this resolution is only one of the possibilities: thanks to its numerous citizenships, the diminished seventh chord can lead everywhere.



[**Figure 17.2**: Resolution of the diminished seventh chord into I of C minor.]

If considered an incomplete ninth chord, the diminished seventh chord can always play the role of dominant or secondary dominant, and it is therefore frequently used in authentic cadences and modulations. Consecutive diminished seventh chords may also occur, both in ascending or descending progressions.

In general, the two interlocking tritones of the diminished seventh chord would resolve by contrary or similar motion: when the tritones are augmented fourths, they resolve outward to sixths; if the tritones are diminished fifths, they resolve inward to thirds (see figure 17.3). Moreover, the rule concerning false relations, mentioned in chapter 11, can be frequently neglected when diminished seventh chords are involved.

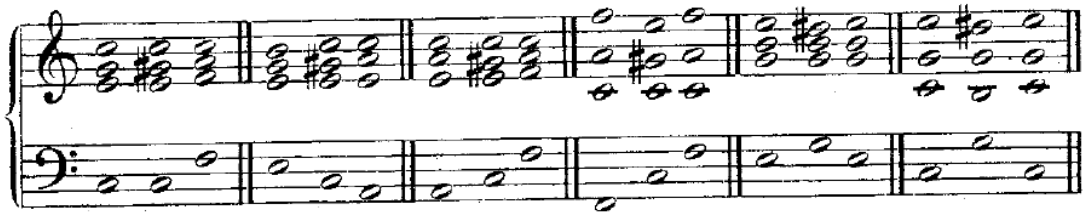


[**Figure 17.3**: Examples of resolution of the diminished seventh chord by enharmonic changing its notes.]

Also the augmented triad can be introduced after almost any chord, and can be successfully used as a secondary dominant. However, it does not have as many resolutions as the diminished seventh chord. In particular, its fifth, a chromatically raised note, should be prepared (that is, it should appear by half-step motion) and resolved (by moving a half-step upward), as shown in figure 17.4. It is very common to prepare the augmented triad with the corresponding unaltered form, i.e. the

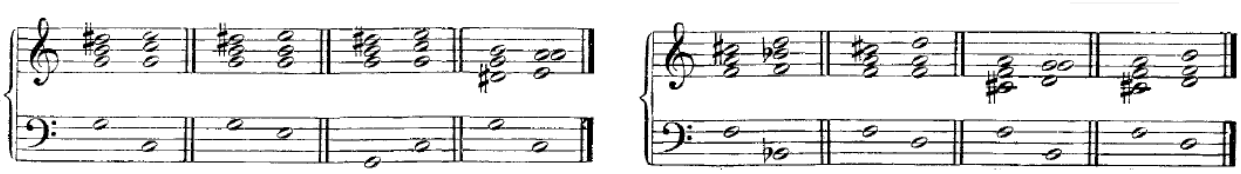
major triad on the same root. In this case, the fifth is the only note that changes: such movement emphasizes its historical origin as embellishment tone (see chapter 14).

Whereas the diminished seventh chord comprehends three dissonant intervals, the augmented triad has only one chromatically altered note. Consequently, its fifth provides a well-definite direction to the chord progressions where the triad is involved. Common resolutions of the augmented triad III of the melodic or harmonic minor scales are I and VI.



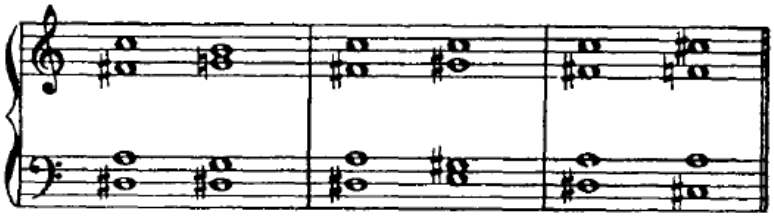
[**Figure 17.4**: Examples of preparation and resolution of the augmented triad.]

The figure 17.5 shows some resolutions for the augmented triad on the dominant and the subdominant degrees of the major scale.



[**Figure 17.5**: Some resolutions of the augmented triad on the fourth and fifth degrees of the major scale.]

In the figure 17.6, some connections of the diminished seventh chord with the augmented triad are shown:



[**Figure 17.6**: Examples of connections of the diminished seventh chord with the augmented triad.]

Bibliography

- Dahlhaus, Carl, "Harmony", *The New Grove Dictionary of Music and Musicians*, vol 8, Macmillan Publishers Limited, London, 1980, pp. 175-188.
- Emery, Stephen A., *Elements of Harmony*, The Arthur P. Schmidt & Co., Boston, 1890.
- Helmholtz, Hermann L. F., *Sensations of Tone*, transl. Alexander J. Hellis, Longmans, Green, & Co., London, 1875.
- Honingh, Aline, *The Origin and Well-Formedness of Tonal Pitch Structures*, ILLC Dissertation Series DS-2006-05.
- Huron, David, *On the Role of Embellishment Tones in the Perceptual Segregation of Concurrent Musical Parts*, Empirical Musicology Review, Vol. 2, No. 4, 2007.
- Prout, Ebenezer, *Harmony: Its theory and practice*, Augener & Co., London, 1889.
- Rameau, Jean-Philippe, *Traité de l'harmonie réduite à ses principes naturels*, Paris: Ballard, 1722, translated as *Treatise on Harmony*, transl. Philip Gossett, New York: Dover Publications, 1971;
- Schoenberg, Arnold, *Theory of Harmony*, University of California Press, Berkeley and Los Angeles, 1978.
- Schoenberg, Arnold, *Structural Functions of Harmony*, W. W. Norton and Company Inc., New York, 1954.
- Tchaikovsky, P.I, *Guide to the practical study of harmony*, transl. Emil Krall and James Liebling, P. Jurgenson, 1900.